Contextual Unification of Classical and Quantum Physics

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Physics ABOUT BROWSE PRESS

Viewpoint: Closing the Door on Einstein and Bohr’s Quantum Debate

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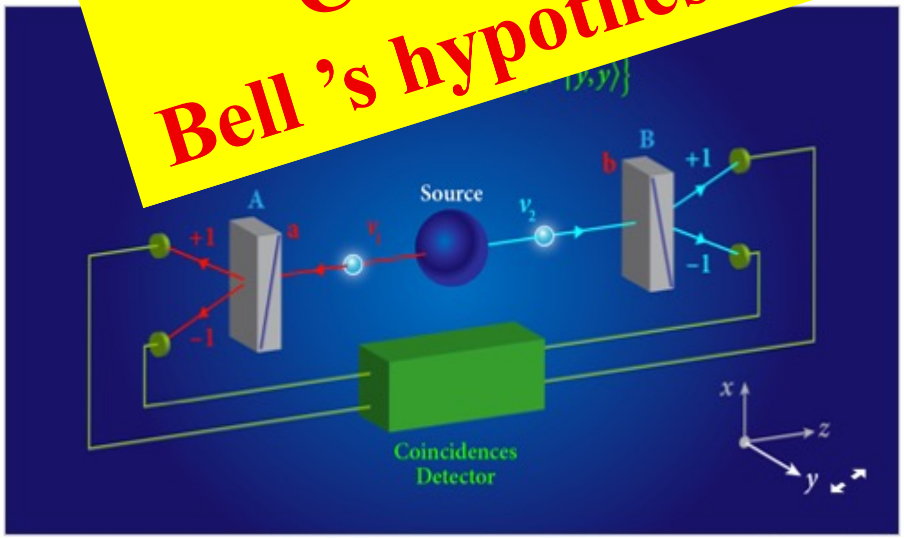
By closing two loopholes at ... remove the last doubts that we should renounc... ...ntum information technologies.

Careful but unavoidable conclusion :
Bell ’s hypotheses (local realism) are untenable !



APS/Alan Stonebraker

B. ... “L... El... M. ... 15). “S... Be... L. ... 15). “S... W. ... 17). “E... sin... les”

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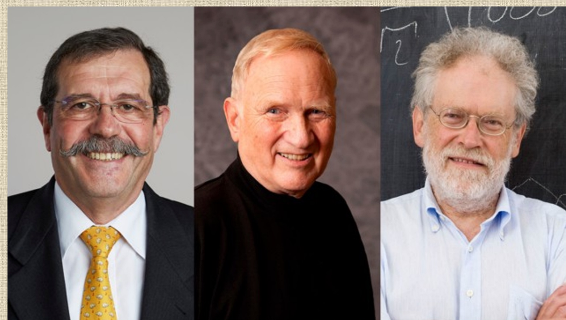


Alain Aspect | John Clauser | Anton Zeilinger

Philosophical standpoint

Many physicists (including me) will support **Physical Realism**, understood as :
The purpose of physics is to study entities of the natural world, existing independently from any particular observer's perception, and obeying universal and intelligible rules.

Many physicists (inc. me) look at **certain and reproducible events as real,** so we like :
If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935)

but Bell tests show that this view does not work as such... **so don't forget Bohr** :
The **very conditions which define the possible types of predictions regarding the future behavior of the system** constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached.

N. Bohr, Phys. Rev. 48, 696 (1935)

What are these « very conditions » required by Bohr to speak about the physical reality of quantum phenomena ?

Element of physical reality vs modality

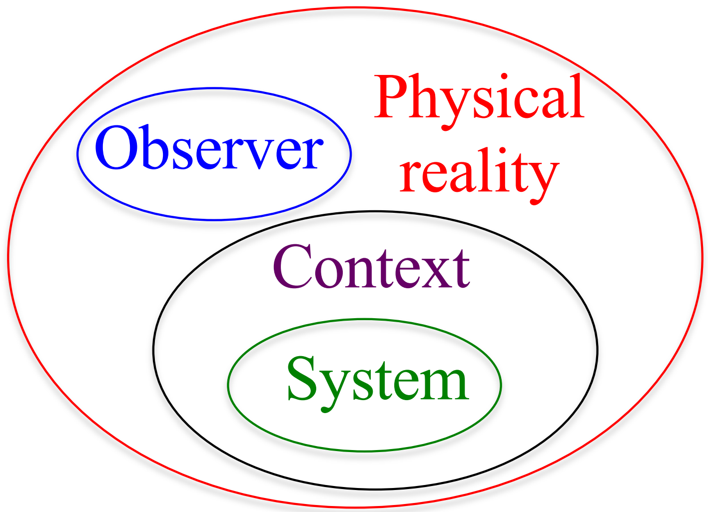
EPR

> If, without in any way disturbing a system we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element physical reality corresponding to this physical quantity.

* This statement agrees with both the **« certainty » required by Einstein and the « very conditions » required by Bohr** to make and to check definite and reproducible predictions (i.e. with objectivity, taken as contextual).

> * Therefore **the « object »** carrying the element of physical reality **is a system within a context.**

* The « split » between system and context is not a problem for CSM, because a modality is defined in terms of both the system and the context, and the system **cannot include the context.**

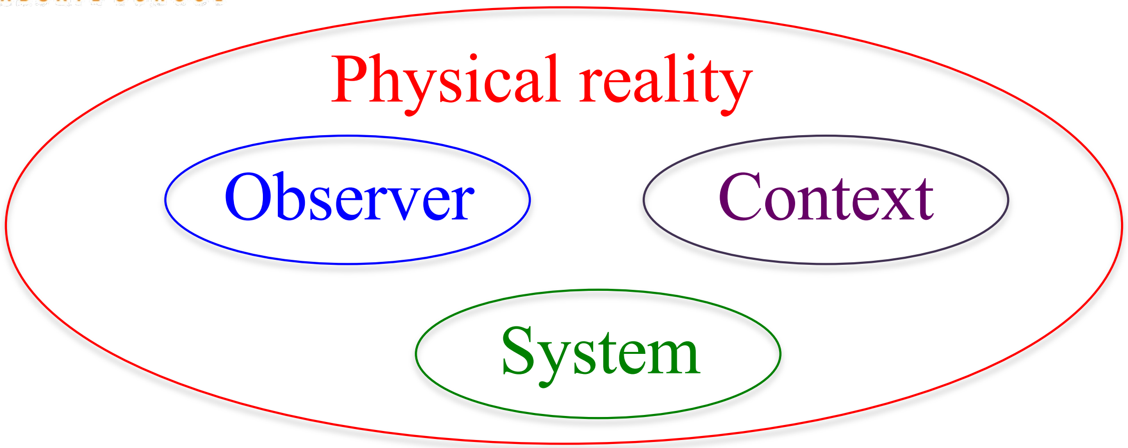


« Although it can describe *anything*,
a quantum description cannot include *everything*. »

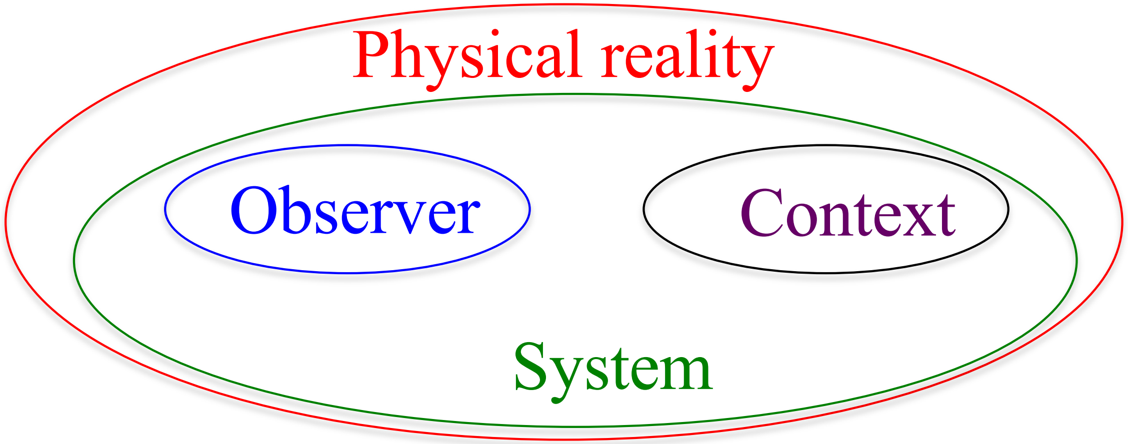
A. Peres and W. H. Zurek, Am. J. Phys. 50, 807 (1982)

Some ontology...

Found. Phys. 46, 121 (2016)
arxiv:1409.2120

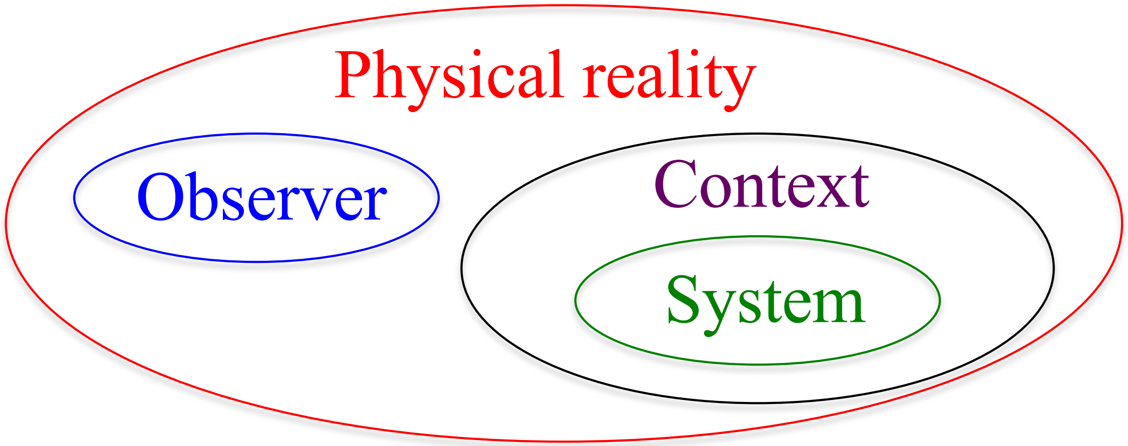


Classical ontology :
the observer can know the "real" physical properties of the system, and the context is only used as an auxiliary tool for measurements.



Usual quantum ontology : through successive "entangling" interactions and unitary evolution, the system will include the context, and also (ultimately) the observer.

Many macroscopic 'realities' ?



CSM ontology : the context appears always between the system and the observer, and definite values of the relevant physical properties (modalities) are attributed jointly to the system and the context.

Unique macroscopic reality !

Contexts, Systems & Modalities: CSM axioms

Axiom 0 (unicity of the macroscopic world)

There is a unique macroscopic physical world where a given measurement yields a single result.

Axiom 1 (modalities)

(i) Given a physical system, a **modality** is defined as the values of a complete set of physical quantities that can be **predicted with certainty and measured repeatedly** on this system.

(ii) Here "complete" means the largest possible set compatible with certainty and repeatability, for all possible modalities attached to this set. **This complete set of physical quantities is called a context, and a modality is attributed to a system within a context.**

(iii) Modalities in different contexts may be connected with certainty (**extracontextuality**)

Axiom 2 (contextual quantization)

(i) For a given context, there exist N distinguishable modalities, that are mutually exclusive: if one modality is true, or realized, the others are wrong, or not realized.

(ii) The value of N, called the dimension, is a characteristic property of a given quantum system, and is the same in all relevant contexts.

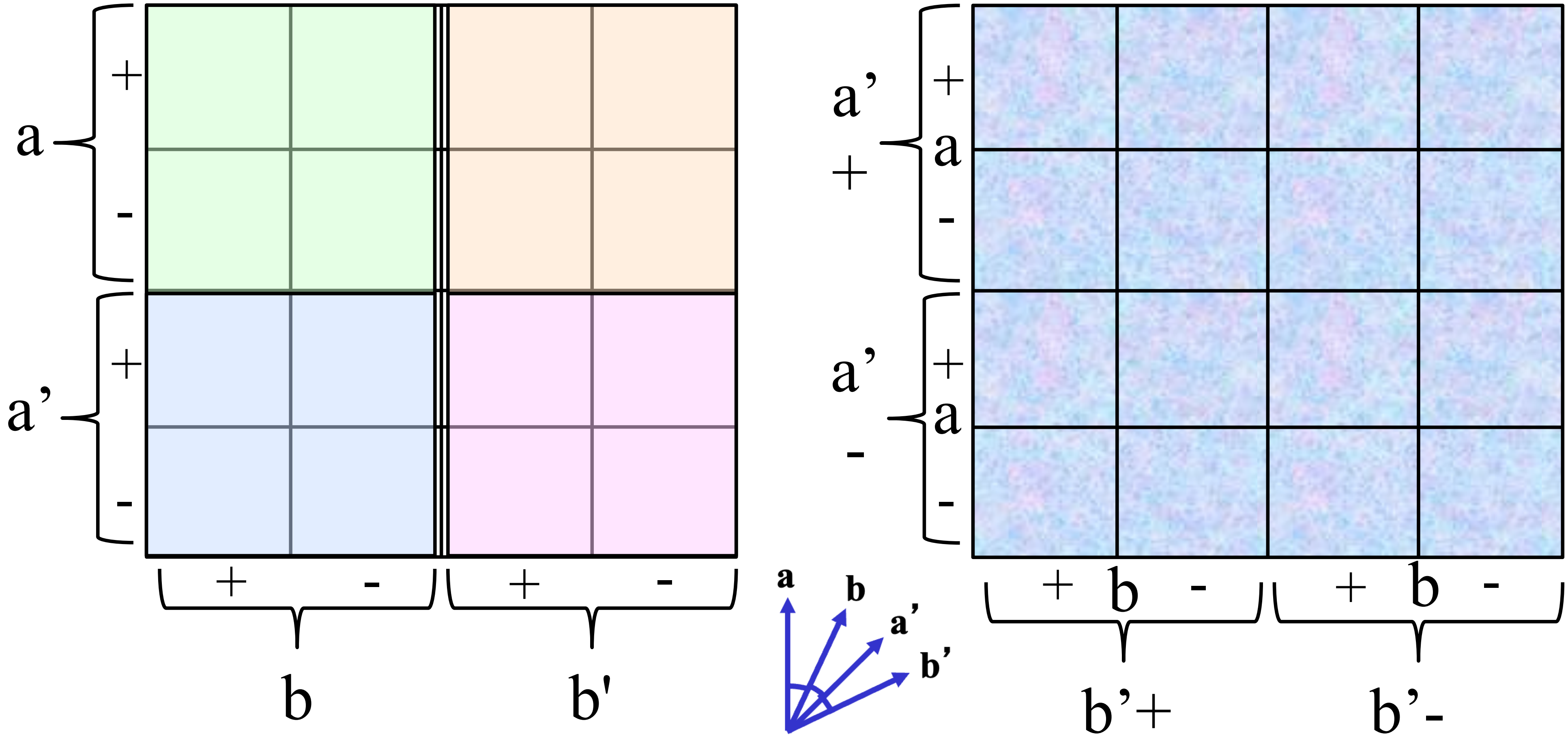
Axiom 3 (changing contexts)

Given axioms 1 and 2, the different contexts relative to a given quantum system are related between themselves by continuous transformations which are associative, have a neutral element (no change), and an inverse. Therefore **the set of context transformations has the structure of a continuous group, which is generally non-commutative.**

Modalities in a Bell experiment

4 different contexts : MQ

Global context : classical



N = 4 mutually exclusive modalities in each context (beware: dichotomic results)

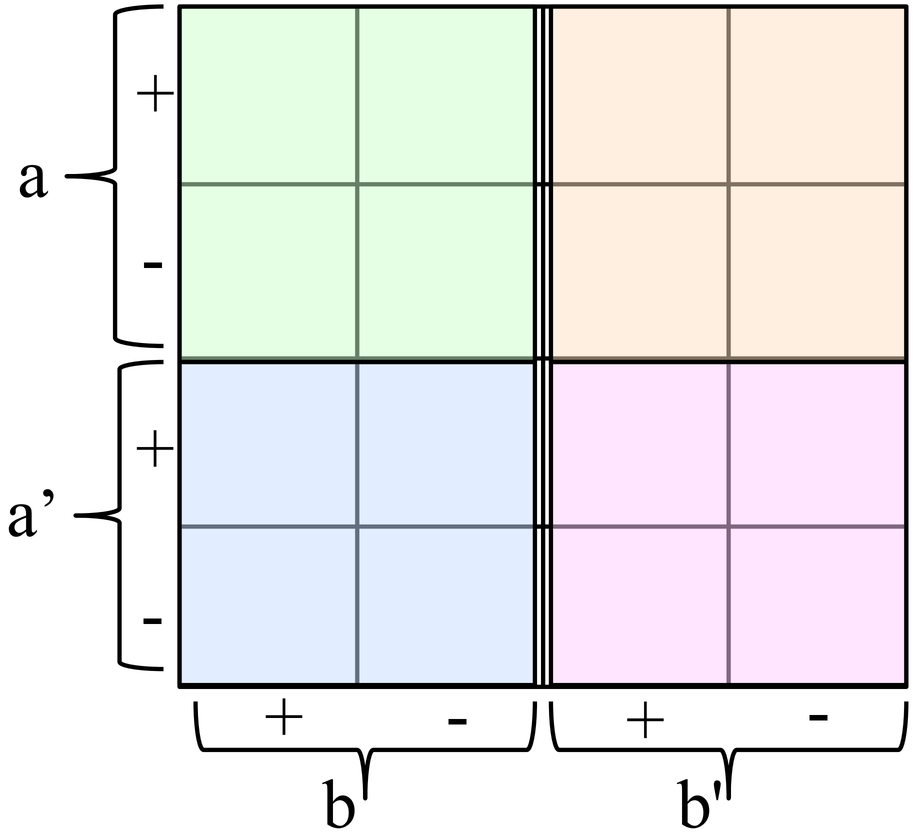
Violation of Bell's ineq. : agreement with expts !

16 mutually exclusive results in a global context

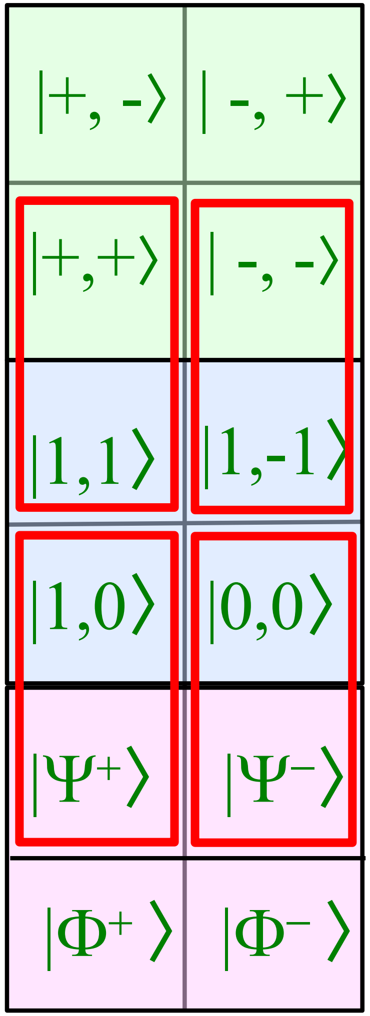
Obeys Bell's ineq. : contradiction with expts !

Modalities in a Bell experiment

4 different contexts : MQ



4 other different contexts : MQ



S_{z1}, S_{z2}
$m_1 = \pm 1/2$
$m_2 = \pm 1/2$

Total spin $\mathbf{S^2}$, S_z
$S = 0,1$
$m = -1,0,1$

Bell states
$|\Phi^{\pm}\rangle = (|+,+\rangle \pm |-,-\rangle)/\sqrt{2}$
$|\Psi^{\pm}\rangle = (|+,-\rangle \pm |-,+\rangle)/\sqrt{2}$

Crucial observation : The certainty of a modality can be transferred between different contexts !

Mutual certainty of modalities is called extravalence (equivalence relation) and the probability belongs to the extravalence class, not to the modality.

Born's rule : the CSM way (1)

Now forget QM, and ask : how can we make sure that
- there are only N mutually exclusive modalities in any context
- the certainty of a modality can be transferred between contexts

Inductive reasoning : use projectors !

Let's attribute a N x N projector to an extravalence class, with
- orthogonal projectors ⇔ mutually exclusive modalities (in a context)
- same projector ⇔ mutually certain modalities (in an extravalence class)

- the probability to find a given result (reproducible with certainty after being found) given an initial 'state' is a function $f(P_n)$, where f depends only on the initial state, and $P_n = |\psi_n\rangle\langle\psi_n|$ is a projector associated with the result.

- the probabilities are additive for mutually orthogonal (commuting) projectors, and $\sum_n f(P_n) = 1$ for any orthogonal set such that $\sum_n P_n = Id$

Born’s rule : the CSM way (2)

Alexia Auffèves & Philippe Grangier, Entropy 24, 199 (2022)
https://arxiv.org/abs/2111.10758

Deductive reasoning : recovering the usual QM formalism

- Theorem (Uhlhorn) : unitary transformations between contexts.

Consider two contexts Cp (with N mutually orthogonal projectors Pi), Cq (with N mutually orthogonal Qj). Mapping the Pi on the Qj when changing the context **must preserve the orthogonality of the projectors**: then it must be a unitary or antiunitary transformation (Uhlhorn’s theorem).

We want also to connect continuously the context change with the identity (no change of context, Cp = Cq) : **unitary transformation only.**

- Theorem (Gleason) : Born’s rule.

The previous requirements fit with the hypotheses of Gleason’s theorem :

- if the probability 1 is reached when changing contexts then one gets

  Born’s rule for pure states, p(j | i) = Trace(Pi Qj).

- otherwise one gets Trace(ρ Qj) where ρ is a density matrix.

Completing the (usual) quantum formalism

1/ A projector $| \psi \rangle\langle \psi |$ **does not define a modality** but an extravalence class, so to make physical sense of the QM formalism one needs

• a state (vector) $| \psi_n \rangle$ or projector $| \psi_n \rangle\langle \psi_n |$

AND

• an observable (operator) $\sum_k a_k | \psi_k \rangle\langle \psi_k |$ with $| \psi_n \rangle \in \{| \psi_k \rangle\}$

Both of them are needed to define a physical modality and to get actual probabilities over a set of mutually exclusive events.

* It can be said that the usual $| \psi \rangle$ is predictively incomplete ; see
P. Grangier, Entropy 23 (12),1660 (2021) https://arxiv.org/abs/2012.09736
Contextual inferences, nonlocality, and the incompleteness of quantum mechanics
* Warning: a PVM is required for reproducibility, not a POVM !

From Kolmogorov to Gleason

P. Grangier, "Kolmogorovian Censorship, Predictive Incompleteness, and the Locality Loophole in Bell Experiments " Entropy 28(1), 80 (2026) [arXiv:2405.03184]

The Kolmogorov axioms are the formal basis of classical probabilities. They define :

1/ A space of events F, which are subsets of a sample set Ω. The space F may be :

- **a Boolean algebra:** finite logical combinations of events
- **a σ-algebra (tribu) :** countably closed extension required for measure theory
- **a Borel σ-algebra :** canonical choice when topology is present

2/ Probabilities, that are measures on F assigning to each event its probability P(A), which is

1. **Positive: $P(A) \geq 0$.**
2. **Normalized: $P(\Omega) = 1$.**
3. **Countably additive:** For any countable sequence of pairwise disjoint events $(A_n)_{n\geq 1}$

$$P\left(\bigcup_{n\geq 1} A_n\right) = \sum_{n\geq 1} P(A_n).$$

Do these axioms apply for Quantum Mechanics ?
This is a rather controversial question, many diverging papers…

From Kolmogorov to Gleason

P. Grangier, "Kolmogorovian Censorship, Predictive Incompleteness, and the Locality Loophole in Bell Experiments " Entropy 28(1), 80 (2026) [arXiv:2405.03184]

Approximate consensus:

- The axioms of positivity, normalization, and countable additivity for disjoint events are still true, **the difficulty is with the event space.**
- The Kolmogorov probabilities do apply within each context (Kolmogorovian Censorship)
- **The problem is "gluing" all the contexts together,** because the global event space in no longer a σ-algebra.

Solution (à la CSM): Replace the classical σ-algebra with a projection lattice, and disjoint additivity with orthogonal additivity => Gleason's hypothesis !

A single measurement context corresponds to a maximal commuting family of projections: restricted to that commuting subalgebra, the projection lattice is isomorphic to a classical Boolean algebra and the probabilities are Kolmogorovian.

Operationally, Kolmogorovian Censorship and Gleason's hypotheses are therefore complementary: KC explains why probabilities look classical inside a fixed context, while Gleason characterizes when and how those context-by-context classical assignments can be coherently extended to a single quantum probability law on the whole projection lattice.

Completing the (usual) quantum formalism

1/ A projector $| \psi \rangle\langle \psi |$ **does not define a modality** but an extravalence class, so to make physical sense of the QM formalism one needs

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2/ But then the formalism should be able to describe **both** the quantum system and the classical context, i.e. **both sides of the (in)famous « Heisenberg cut »**. **How to do that ?**

About infinite tensor products.

M. Van Den Bossche & P. Grangier, Found. Phys. 53:45 (2023)
Proc. DICE conf. (2023), Entropy 25, 1600 (2023)

* Composite systems are described using tensor products as usual.

* **Contexts = infinite tensor product ?** Taking this limit breaks unitarity, and leads to sectorization in type III algebra (see : von Neumann 1939, "On infinite direct products").

Naively, one would expect to get an "infinitely large Hilbert space", still with the same algebraic properties, but this turns out to be completely wrong.

Quoting von Neumann*: *Infinite (tensor) products differ essentially from the finite ones in this, that they split up into "incomplete tensor products". (...) What happens could be described in the quantum-mechanical terminology as a splitting up of the tensor product into "non-intercombining systems of states", corresponding to the incomplete direct products quoted above."*

* *J. von Neumann, Compositio Mathematica* 6, 1-77 (1939)

Modelling macroscopic quantum systems

- **Statistical physics**
  - Systems of N microscopic elements
  - $N \rightarrow \infty$: macroscopic / thermodynamical limit
  - Ex. : non-integer power law correlation functions (non analytic) in critical phenomena $\langle \boldsymbol{S}(x) . \boldsymbol{S}(y) \rangle \approx | x - y |^{-\beta}$, β non integer : *need to take this limit.*

  In StatPhys $N \rightarrow \infty$ is a *valid model* to represent real, macroscopic systems.

- **Quantum physics**
  - Microscopic element α described using a Hilbert space H_α
  - The dimension $d_\alpha > 1$ of H_α is finite (qudit…) or countably infinite (square integrable wave functions)
  - N microscopic elements are described by $H = H_1 \otimes H_2 \otimes \ldots \otimes H_N$
  - **It is reasonable to assume macro systems are described by a $N \rightarrow \infty$ limit**

    => Interesting to understand what are the properties of such a H_∞

Spoiler : one should expect exotic properties

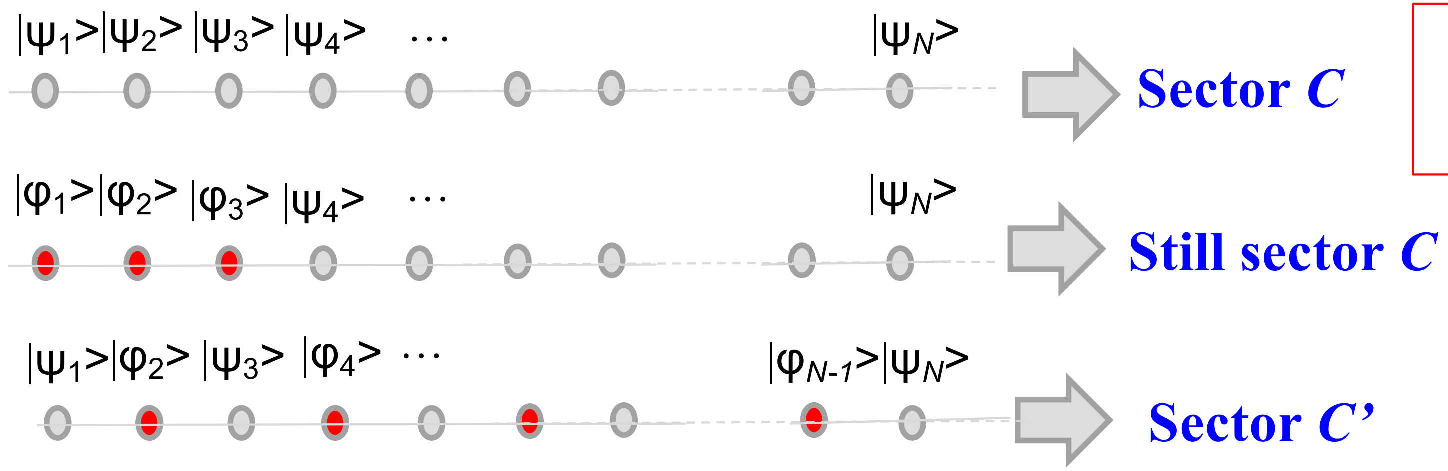
e.g. for qubits: $d_\alpha \equiv 2$, $\dim(H_\infty) = 2^{\aleph_0} = \aleph_1$ i.e. the power of continuum.

=> There is no countable basis dense in H_∞

=> H_∞ is not separable – different from what is used in textbook QM

Quantum states of macroscopic systems

- **The tensor product of an infinity of Hilbert spaces $H = \otimes_\alpha H_\alpha$ decomposes into the direct sum of orthogonal « sectors »**
  - In a sector, vectors differ by a finite (microscopic) number of components
  - Between sectors, vectors differ by an infinite (macroscopic) number of components
  - **Sectors are separable Hilbert spaces, and they are in uncountable number**
  - **Sectors are in direct sum and together they generate the full Hilbert space**



$$\bigotimes_{i=1}^{\infty} \mathcal{H}_i = \bigoplus_{\alpha \in A} \mathcal{H}^{(\alpha)}$$

* Under a change of states affecting only a finite number (a microscopic fraction) of the degrees of freedom, the macroscopic system remains in the same sector

* Under a change of states affecting an infinite number (a macroscopic fraction) of the degrees of freedom, the macroscopic system moves from a sector to an orthogonal one.

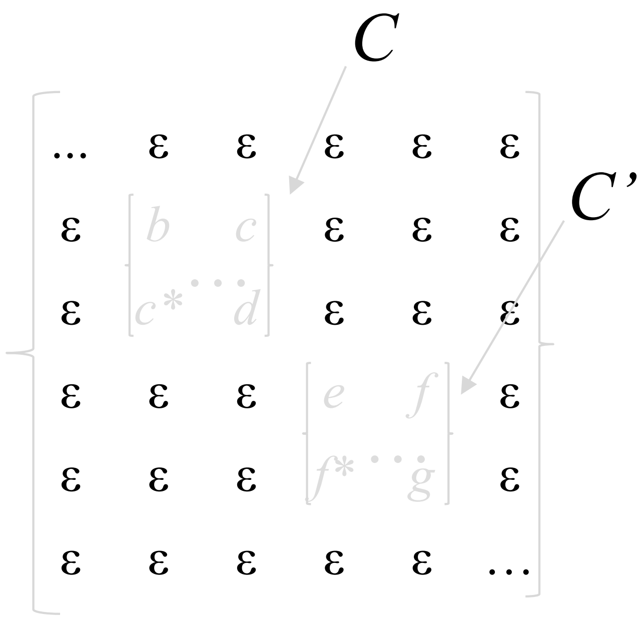
Sectorization and self-decoherence

Sectorization for states and operators

- **Superposition of macroscopically different states (in different sectors) cannot be built.**
- **Operators built from elementary operators commute with projectors on sectors**

At finite *N*, blocks start building up,

with $\varepsilon \to 0$ as $N \to \infty$



$$\bigotimes_{i=1}^{\infty} \mathcal{H}_i = \bigoplus_{\alpha \in A} \mathcal{H}^{(\alpha)}$$

In the N → ∞ limit, sector blocks

- **have countably infinite dimension**
- **are in uncountably infinite number**
- **have no quantum correlation among one another**

=> Sectors are the continuum of classical states of the macro system

=> For finite *N*, there are residual correlations between will-be sectors

The breakdown of the Hilbert space causes a self-decoherence of the system in the large *N* limit

And what about BEC, superconductors/superfluids, Fermi liquids, etc?

→ The relevant degrees of freedom are quasiparticules, and Fock space is separable.

CSM construction: universality and completeness

Found. Phys. 51, 76 (2021) http://arxiv.org/abs/2003.03121

M. Van Den Bossche & P. Grangier, https://arxiv.org/abs/2209.01463

* Using a sectorized global algebra provides a complete description corresponding to the modalities, and not to the usual $|\psi\rangle$ describing an extravalence class: ok. **The algebra is universal, but there is no universal wavefunction (it is 'split up')**

*** On this basis, unitary evolution is a feature of properly isolated subsystems, and there is no universal unitarity in quantum theory, but an algebraic description including explicitly the loss of unitary equivalence.**

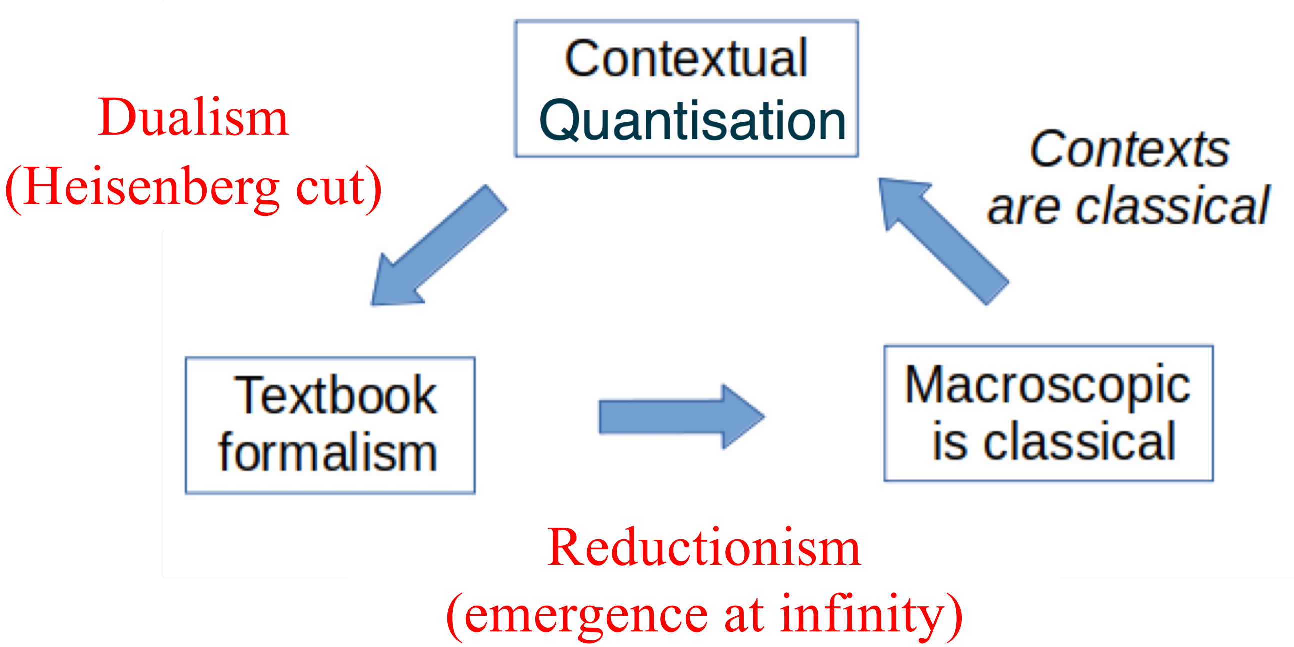
* Important point : there is no need to specify all details for the context (this is not possible : there are « infinitely many » details), and it is enough to label the different sectors by using the **commutative 'center' of the type III algebra**. This is just what is needed for a classical description of the context.

* There were related works during the 1970's e.g. by Hepp, Araki, Emch, Bub…. but they have been superseded by the decoherence approach (Zeh, Zurek et al), which is not considered as fully satisfactory either. **So why looking at this again ?**

Closing the loop of CSM

Found. Phys. 51, 76 (2021) https://arxiv.org/abs/2209.01463

Entropy 25, 1600 (2023). https://arxiv.org/abs/2310.06099



* **CSM links dualism and reductionism,** so one implies the other in a closed loop. This means that both options can be viewed as ***equivalent*** (and not antagonistic).

*** Can we use infinities in a physical theory ? Two arguments to say yes:**

On the mathematical side, the non-separable, sectorized limit builds up gradually, at least in the weak topology relevant for von Neumann algebras (work in progress)

On the epistemological side, representations and reality are of different nature, and **conceptual elements of a model are not elements of reality.**

What next ?

Some obvious questions (ongoing work!) :

- **Dynamics: can we describe what happens « during a measurement » ?**
- **Even if legitimate, infinity is far away, can we tell how to « approach » it ?**

Dynamics: replace the usual Hamiltonian evolution by the « modular flow » (Tomita – Takesaki) which depends also from a reference state ω :

Heisenberg Equation, physical time t

$$A(t) = e^{\,iHt/\hbar} A \; e^{-iHt/\hbar} \;\; \Rightarrow \;\; \sigma_\tau^{\omega}(A) = \Delta_\omega^{\,i\tau} \; A \; \Delta_\omega^{-i\tau}$$

Modular operator Δ_ω , modular time τ

During a measurement some operators become « outer » and are not any more in the accessible algebra: this insures an intrinsic irreversibility.

Modular time: the modular operator Δ depends on the reference state ω (quite unusual !) and the (dimentionless) modular time is different from the physical time. Suitable reference state ω ?

=> KMS state: infinite tensor product of Gibbs states $\rho = \exp(-H / kT) / Z$

=> Thermal time $t_{th} = \hbar / kT$ and modular time $\tau = t / t_{th}$

What next ?

Some obvious questions (ongoing work!) :

- **Dynamics: can we describe what happens « during a measurement » ?**
- **Even if legitimate, infinity is far away, can we tell how to « approach » it ?**

Approaching infinity: the behaviour is in general model-dependant.

For instance, consider a context (apparatus) made of an infinite spin chain at temperature T, with an Ising coupling $H = \sum_i J\, Sz_i\, Sz_{i+1}$ between the spins.

This creates a correlation length $Nc = 1/\log(\coth(J/kT))$ between the spins.

Then the predictions obtained from a chain of length N are exponentially close to those obtained from the modular flow, by terms in $\exp(-N/Nc) \ll 1$ for long chains.

Open questions:

- Is there a special role of type III_1 algebras, associated with KMS states ?
- Unicity from ergodicity ?

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Thank you for your attention !

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For an informal introduction to (CSM) quantum physics, see:

The two-spin enigma: from the helium atom to quantum ontology

https://www.mdpi.com/1099-4300/26/12/1004

Thank you to Franck Laloë, Roger Balian, Karl Svozil...

Some references on Quantum Foundations

* Another view on QM: Contexts, Systems and Modalities (CSM)

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Deriving Born's rule from an Inference to the Best Explanation

A. Auffèves & P. Grangier, Entropy 24 (2), 199 (2022) arxiv:2111.10758

Revisiting Born's rule through Uhlhorn's and Gleason's theorems

* Another view on Bell's theorem: ψ is predictively incomplete

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Contextual inferences, nonlocality, and the incompleteness of quantum mechanics

* From John von Neumann (1939) to operator algebras

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Contextual unification of classical and quantum physics

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Postulating the Unicity of the Macroscopic Physical World