Entanglement and phase correlations. A quantum field theory approach *

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*M.Blasone, P.Jizba, G.Vitiello, Quantum field theory and its macroscopic manifestations, Imperial College Press 2011
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- **Entanglement: experimentally well established phenomenon.**

- **The suspicion that "spooky action at a distance" associated with entanglement might violate special relativity (Einstein, Podolsky, Rosen experiment (1935)) is ungrounded.**

- **David Bohm: hidden variables theory (1952).**

- **Bell's theorem (1964): QM results cannot be explained by any** *local* **hidden variables theory.**

- **An enormous list of references discussing many theoretical and experimental points. Practically all of them within the frame of QM.**

- **A quantum field theory approach:**

an attempt to get a somewhat more concrete and intuitively graspable view of entanglement.

In QFT, for two entangled photons with opposite polarization,

the correlation described as entanglement is actually imprinted in the structure of the vacuum.

it is 'read' from this structure in the vacuum expectation values of operators of the two photons.

- A characterizing property of QFT (non-existing in QM, the von Neumann theorem):

the existence of
infinitely many unitarily non-equivalent representations
of the canonical (anti-)commutation relations (CCR).

The standard QFT formalism, called the LSZ (Lehmann, Symanzik, Zimmerman) formalism develops on two levels,
- the one of the dynamics for the interacting or Heisenberg fields and
- the one of the asymptotic or physical fields.

Quantum Field Theory

two levels

Dynamical fields

$\psi(x, t)$

Non-linear dynamical Eqs.

Physical Fields

$\phi(x, t)$

Phenomenology

linear Eqs.

$$\psi(x, t) \quad = \quad F\,[\phi(x, t)]$$

dynamical map

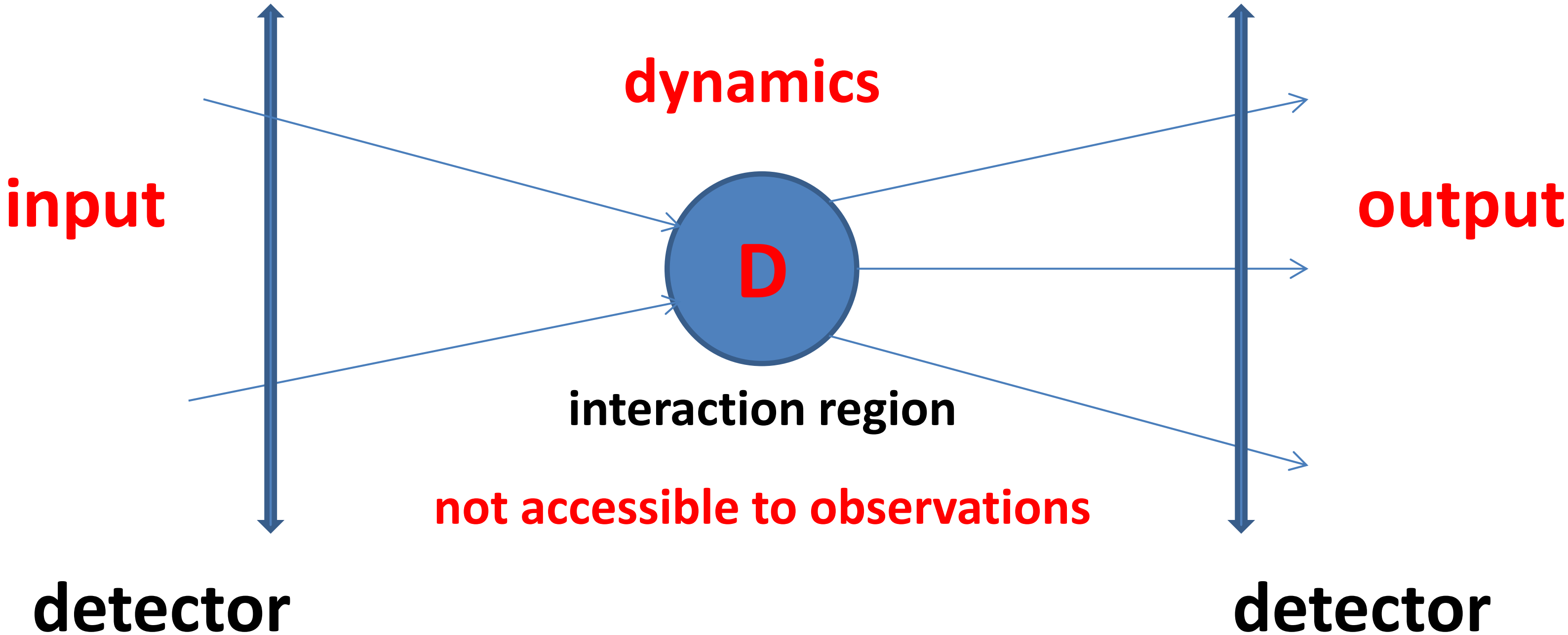
"week equality" between

physical states

- **The interaction region is not accessible to observations**

- **Observations are possible only in the asymptotic regions**

Observables are described in terms of asymptotic fields $\phi(x, t)$.


dynamics
input
D
output
interaction region
not accessible to observations
detector
detector

- The space $\mathcal{H}_{\mathcal{F}}$ of the physical states is the only accessible state space.

The physical states are by definition the ones where the total energy and the total momentum are given by the sum of the energy and of the momentum, respectively, of each constituent particle.

- The Heisenberg fields are expressed in terms of asymptotic fields through the dynamical map

$$\langle b|\psi(x,t)|a\rangle = \langle b|F[\phi(x,t)]|a\rangle \tag{1}$$

for any $|a\rangle$ and $|b\rangle$ belonging to the space $\mathcal{H}_{\mathcal{F}}$ of the physical states.

Any relevant dynamical quantity, for example the Hamiltonian $H(\psi)$ is thus expressed in terms of the asymptotic fields $\phi(x,t)$ by weak equalities (denoted by the symbol "$\stackrel{w}{=}$"):

$$H(\psi) \stackrel{w}{=} H(F[\phi]) \equiv H_0(\phi) + W_0, \tag{2}$$

W_0 is a constant commuting with any operator.

The Hamiltonian associated to the physical states provides of course the total energy of the asymptotic particles detected by the observers.

Thus the specific form of H_0 is the one of a free Hamiltonian for the asymptotic particles.

However, this does not mean that the interaction described by H has been neglected.

It is instead fully taken care by the dynamical nature of the map $\psi^{\ w} = F[\phi]$.

Thus, the "non-interacting" asymptotic outgoing fields actually carry in their quantum numbers, energy, spin, charge, etc. the memory of the interaction out of which they are produced.

Such a memory is imprinted in the physical vacuum and other physical states.

• We consider the simultaneous creation of two photons by a neutral spin-zero source, as in the usual parametric down-conversion process.

In the source rest frame (centre-of-momentum frame): the photons have opposite momenta k and $-\mathrm{k}$ and energy $\omega_k \approx (1/2)\omega_p$, ω_p is the source energy.

The usual parametric down-conversion term Hamiltonian in nonlinear media optics is

$$H = \sum_{\mu} \int d^3k \left[\omega_k (a^{\dagger}_{\mu,\mathbf{k}} a_{\mu,\mathbf{k}} + b^{\dagger}_{\mu,\mathbf{k}} b_{\mu,\mathbf{k}}) + \nu_k (b^{\dagger}_{-\mu,-\mathbf{k}} a^{\dagger}_{\mu,\mathbf{k}} + a_{\mu,\mathbf{k}} b_{-\mu,-\mathbf{k}})\right], \tag{3}$$

a and b denote the operators for the two photons in the pair;
$\mu = \pm 1$ is the spin index.

They are the interacting or Heisenberg field operators.

ω_k and ν_k are real positive (hermiticity of H). Also, it is $\nu_k^2 < \omega_k^2$.
Usual commutation relations for the a and b operators.

The vacuum state for the two photons is $|0\rangle \equiv |0\rangle_a \otimes |0\rangle_b$

$$a_{\mu,\mathbf{k}}|0\rangle = 0, \qquad b_{\mu,\mathbf{k}}|0\rangle = 0, \tag{4}$$

and $\langle 0|H|0\rangle = 0$.

$|0\rangle$ belongs to the Hilbert space $\mathcal{H}$ which is NOT the physical state space for the observers.

Alice and Bob have no access to the interaction region.
Their observation can only occur in the asymptotic region, where the Hamiltonian, written in terms of asymptotic fields α and β has the form

$$H = H_0 + W_0 = \sum_\mu \int d^3k E_k (\alpha^\dagger_{\mu,\mathbf{k}}\alpha_{\mu,\mathbf{k}} + \beta^\dagger_{\mu,\mathbf{k}}\beta_{\mu,\mathbf{k}}) + W_0 \ , \tag{5}$$

Equalities are weak equalities. E_k is the total energy of the α-modes and β-modes.

The Hamiltonian is transformed in this form by using the dynamical maps relating asymptotic operator fields with interacting photon fields

$$\alpha_{\mu,\mathbf{k}}(\theta) = a_{\mu,\mathbf{k}} \cosh\theta_k - b^{\dagger}_{-\mu,-\mathbf{k}} \sinh\theta_k \tag{6}$$

$$\beta_{\mu,\mathbf{k}}(\theta) = b_{\mu,\mathbf{k}} \cosh\theta_k - a^{\dagger}_{-\mu,-\mathbf{k}} \sinh\theta_k \tag{7}$$

We recognize that they are the Bogoliubov transformations with the phase θ_k.

The vacuum state for these asymptotic photon modes is NOT $|0\rangle$, but the physical vacuum $|0(\theta)\rangle$, $\theta \equiv \{\theta_k, \forall\ k\}$:

$$|0(\theta)\rangle = \exp\left(-\frac{V}{(2\pi)^3}\int d^3k\ \ln\cosh\theta_k\right)\exp\left(\int d^3k\ \tanh\theta_k\, a^{\dagger}_{\mu,\mathbf{k}} b^{\dagger}_{-\mu,-\mathbf{k}}\right)|0\rangle. \tag{8}$$

$$\alpha_{\mu,\mathbf{k}}|0(\theta)\rangle = 0, \qquad \beta_{\mu,\mathbf{k}}|0(\theta)\rangle = 0. \tag{9}$$

We have

$$E_k = \sqrt{\omega_k^2 - \nu_k^2}\ , \qquad W_0 = \int d^3k \left[\sqrt{\omega_k^2 - \nu_k^2} - \omega_k\right]\ , \tag{10}$$

and

$$\cosh 2\theta_k = \frac{\omega_k}{\sqrt{\omega_k^2 - \nu_k^2}}\ , \qquad \sinh 2\theta_k = -\frac{\nu_k}{\sqrt{\omega_k^2 - \nu_k^2}}\ . \tag{11}$$

$|0(\theta)\rangle$ **is a generalized** $SU(1,1)$ **two-mode squeezed coherent state; the phase** θ_k **is related to the squeezing parameter.**

In the infinite volume limit, $\mathcal{H}$ **is unitarily inequivalent to** $\mathcal{H}_{\mathcal{F}}(\theta)$**, for any** $\theta \equiv \{\theta_k, \forall\ k\}$**, i.e.**

$$\langle 0|0(\theta)\rangle \to 0 \quad \text{for} \quad V \to \infty\ , \tag{12}$$

and similarly for any couple of states of $\mathcal{H}$ **and** $\mathcal{H}_{\mathcal{F}}(\theta)$**;**

Note that, while

$$\lim_{V\to\infty}\lim_{\theta\to 0}|0(\theta)\rangle = |0\rangle, \tag{1}$$

it is

$$\lim_{\theta\to 0}\lim_{V\to\infty}|0(\theta)\rangle \neq |0\rangle, \tag{2}$$

the Haag theorem*.

$|0(\theta)\rangle \perp |0\rangle$ **because we miss contributions of order of $1/V$ in the infinite volume limit ("locality").**

$\Rightarrow$ **non-perturbative physics!!**

note that $|0(\theta)\rangle \perp |0(\theta')\rangle$**, for any** $\theta \neq \theta'$**.**

there are (infinitely) many unitarily inequivalent (physically different) $\mathcal{H}_{\mathcal{F}}(\theta)$ **for all** θ**.**

* **N. N. Bogoliubov, A. A. Logunov, I.T. Todorov,** ***Axiomatic Quantum Field Theory*****, Benjamin, New York, 1975.**

We have:

- $|0(\theta)\rangle$ **is an entangled state for the photon pair** $a_{\mu,\mathbf{k}}$ **and** $b_{-\mu,-\mathbf{k}}$**:**

$$|0(\theta)\rangle = \prod_k \frac{1}{\cosh\theta_k}\left[|0\rangle\otimes|0\rangle + \sum_{\mathbf{k},\mu}\tanh\theta_k\left(|a_{\mu,\mathbf{k}}\rangle\otimes|b_{-\mu,-\mathbf{k}}\rangle\right) + \ldots\right], \quad (15)$$

it cannot be factorized into the product of two single-mode states.

- **A measure of the degree of entanglement is provided by the entropy**

$$\mathcal{S}_a \equiv -\sum_{\mathbf{k},\mu}\{a^\dagger_{\mu,\mathbf{k}}a_{\mu,\mathbf{k}}\ln\sinh^2\theta_k - a_{\mu,\mathbf{k}}a^\dagger_{\mu,\mathbf{k}}\ln\cosh^2\theta_k\} \quad (16)$$

$$\langle 0(\theta)|\mathcal{S}|0(\theta)\rangle = -\sum_{n=0}^{+\infty} W_n(\theta)\ln W_n(\theta)\ . \quad (17)$$

$$W_n(\theta) = \prod_{k,\mu}\frac{\sinh^{2n_{\mu,k}}\theta_k}{\cosh^{2(n_{\mu,k}+1)}\theta_k}, \quad 0 < W_n < 1; \quad \sum_{n=0}^{+\infty} W_n = 1 \quad (18)$$

The state $|0(\theta)\rangle$ is

$$|0(\theta)\rangle = \sum_{n=0}^{+\infty} \sqrt{W_n}|n, n\rangle, \tag{19}$$

n **denotes the set** $\{n_{\mu,k}\}$.

W_n **gives the probability of having entanglement of the two sets of** $\{n\}$ **modes** a **and** b**,**

- **In the summation term in $|0(\theta)\rangle$ there are states of the Bell type, e.g.**

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\Big(|+, 1_{\mathbf{k}}\rangle\, |-, 1_{-\mathbf{k}}\rangle \pm |-, 1_{\mathbf{k}}\rangle|+, 1_{-\mathbf{k}}\rangle\Big)\,, \tag{20}$$

- W_n **is a decreasing monotonic function of** n**. The entanglement seems to be suppressed for large** n

However, it is effectively realized only when the summation extends to an infinite number of components, in the QFT infinite volume limit,

so that *phase transition* **to** $\mathcal{H}_{\mathcal{F}}(\theta)$**, the "entanglement phase", has effectively occurred**

the robustness of the entanglement arises from the fact that in the QFT infinite volume limit there is no unitary generator able to disentangle the a **and** b **modes.**

• the non-zero contribution to the Alice's measurement of the a-mode number, is actually determined by the non-zero commutator of Bob's β-modes (not of the Alice's α modes!):

$$n_{a_{\mu,k}}(\theta) = \langle 0(\theta)| a^{\dagger}_{\mu,\mathbf{k}} a_{\mu,\mathbf{k}} |0(\theta)\rangle = \langle 0(\theta)| \beta_{-\mu,-\mathbf{k}} \beta^{\dagger}_{-\mu,-\mathbf{k}} |0(\theta)\rangle \sinh^2 \theta_k = \sinh^2 \theta_k .$$

the phase correlation between the two photon modes is *dynamically built in* **in the $|0(\theta)\rangle$ vacuum**

in usual experiment with calcite crystals, if Bob observes the photon in one of the two polarization states, this will determine Alice's observation of the (other) orthogonal polarization photon state, even if their observations occur at a space-like distance.

Such effect is not induced by (spooky) forces, but due to the coherence of the $|0(\theta)\rangle$ condensate.

• **The linear correlation coefficient is found to be**

$$J(\widehat{n}_a, \widehat{n}_b) = \frac{cov(\widehat{n}_a, \widehat{n}_b)}{(\langle(\Delta \widehat{n}_a)^2\rangle)^{1/2} (\langle(\Delta \widehat{n}_b)^2\rangle)^{1/2}} = 1 \ , \tag{21}$$

such a strong (a,b)-pair correlation is due to the coherent structure of the of QFT θ-vacuum,

it is a phase mediated correlation

• **A single measurement made by Bob can determine only a probability distribution for the result observed by Alice.**

We find the Bose-Einstein distribution for the measured number of $a_{\mu,\mathbf{k}}$ (or $b_{\mu,\mathbf{k}}$) photon modes

$$n_{a_{\mu,k}}(\theta) = \sinh^2 \theta_k = \frac{1}{e^{\beta\omega_k} - 1} \ , \tag{22}$$

- θ_k **is actually temperature dependent,** $\theta_k = \theta_k(\beta)$**,**

- **variations of the degree of entanglement, turn into variations of the number of condensate modes.**

- **conserved quantity (**$SU(1,1)$ **Casimir operator):**

$$(a^\dagger_{\mu,\mathbf{k}} a_{\mu,\mathbf{k}} - b^\dagger_{-\mu,-\mathbf{k}} b_{-\mu,-\mathbf{k}})|0(\theta)\rangle = 0 \qquad (3)$$

b **are the "holes" of the** a **modes, or vice-versa (one the "time-reversed image" of the other one (forward in time and backward in time modes).**

Conclusions

What appears at first sight as a "spooky action at a distance" turns out to be the fact that Alice's and Bob's measurements are embedded in the coherent dynamics of the physical vacuum acting as a "collective" mode background.

The correlations between space-like separated modes in the QFT physical vacuum $|0(\theta)\rangle$ are the manifestations of quantum coherence,

by themselves constitute a realistic feature (an "element of reality") of the physical dynamics in a similar fashion as a realistic feature is attributed to the coherent vacuum condensates in crystals, ferromagnets, superconductors, etc..

They are *phase* correlations, not-mediated by messenger particles, and therefore there is no violation of relativity postulates

Applications....

Scalar fields in curved background, near event horizon, Rindler space-time...[*]

Other applications, neutrino mixing and oscillations[†]**, biological systems, neuroscience**[‡]**,**

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Work in progress.

[*] **A. Iorio, G. Lambiase, G. Vitiello, Ann. of Phys. 2001; 2004**

[†] **M. Blasone, P. Jizba, G. Vitiello,** *Quantum field theory and its macroscopic manifestations*, **Imperial College Press 2011**

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Quantum Field Theory and its Macroscopic Manifestations

Boson Condensation, Ordered Patterns and Topological Defects

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