AFSCET

# Modélisation Quantique

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# Mécanique quantique et théorie unitaire

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# QUANTUM PHYSICS REVISITED
(Steven Weinberg)

- Quantum Mechanics Proper :

6 axioms

- Quantum Electrodynamic :

Spin, Photon, Electromagnetism, Atoms and Molecules

- Quantum Theory of Fields :

Elementary particles, nuclear forces, nuclei and transformation of elementary particles

# THE 6 AXIOMS OF QUANTUM MECHANICS

- Physical states of a system are represented by vectors ψ in a Hilbert space H,

defined up to a complex number (a ray in a projective Hilbert space)

- Observables are represented by Hermitian operators
- The result of any physical measure is one of the eigen values λ of the associated operator Φ. After the measure the system is in the state represented by the corresponding eigen vector ψλ
- The probability that the measure is λ is equal to $|\langle \psi\lambda, \psi \rangle|^2$. If a system is in a state represented by a normalized vector ψ , and an experiment is done to test whether it is in one of the states ψn {n=1..N} which constitutes an orthonormal set of vectors, then the probability of finding the system in the state ψn is $|\langle \psi n, \psi \rangle|^2$ .
- When two systems interacts, the vectors representing the states belong to the tensorial product of the Hilbert states
- For the evolution of the system, the Schrödinger's equation.

# THE ISSUES

- The axioms are not related to any specific physical body or property : they could be implemented in any domain
- But the common answer has been to find explanations in some strange properties of the physical world (the “2 physics” separated by a scale, which is not given in the axioms).

- Actually all is about the description of the physical system by a set of mathematical equations and variables

# THE TYPICAL FAMILY OF VARIABLES

GENERAL CONDITIONS 1

- 1) The system is represented by a fixed finite number N of variables Xk, k=1…N
- 2) Each variable belongs to an open subset Ok of a Fréchet real vector space Vk
- 3) At least one of the vector spaces Vk is infinite dimensional
- A state of the system is defined by the value X of the variables Xk which are functions, belonging to a set $O=\prod Ok \subset V=\prod Vk$.
- A Fréchet space is a Hausdorff complete, topological vector space, endowed with a countable family of semi-norms. As a consequence it is metric, normal, completely regular, locally convex, second countable, and T4. Because it is second countable it is separable.
- Example :The space of r differentiable, compactly supported, sections of a vector bundle is an infinite dimensional Fréchet space

# FUNDAMENTAL THEOREMS

- For any system represented by a model meeting the conditions 1, there is a separable, infinite dimensional, Hilbert space H, defined up to isomorphism, such that the set of the variables $X_k$ can be embedded as an open subset of H which contains 0 and a convex subset
- This is the implementation of the Henderson's theorem
- For any basis $e_i, i \in I$ of V contained in O, there are unique families $(\varepsilon_i),(\phi_i), i \in I$ of independent vectors of H, a linear isometry $T:V \rightarrow H$ such that :

$\forall X \in O: T(X) = \sum \langle \phi_i, T(X) \rangle \varepsilon_i \in \Omega$

$\forall i \in I: \varepsilon_i = T(e_i)$

$\forall i,j \in I: \langle \phi_i, \varepsilon_j \rangle = \delta_{ij}$

and T is a compatible chart of the manifold M=(O,X)

- Any real separable infinite dimensional Hilbert space can be endowed with the structure of a complex separable Hilbert space

# OBSERVABLE

- The measure of a variable requires an infinite number of figures. The simplest solution to define an observable YJ is to select a finite number J of vectors of a a basis ei of the vector space V. The map YJ is the projection on the vector subspace VJ spanned by the vectors ei,i∈J.
- To any observable YJ is associated uniquely a self-adjoint, compact, trace-class operator on H : $ZJ=T^{-1}\circ YJ\circ T$ such that the measure of the primary observable YJ, if the system is in the state X∈O, is $YJ(X)=\sum<\phi i,ZJ(T(X))>ei$
- A natural extension of the spaces of these operators leads to commutative von Neuman Algebras. In axiomatic QM it is common to define a system itself by the von Neuman algebra of its observables. And one sees that this algebra is necessarily commutative.

# OBSERVABLE AND PROBABILITY

- The choice of an arbitrary set of vectors of a basis introduces an uncertainty in the measure. It can be computed. For any primary observable $\Phi J$, the value which is measured is an eigen vector of the operator $\Phi J$, and the probability to measure a value $\Phi J(X)$ if the system is in the state X is : $\Pr(\Phi J(X) | X) = \|ZJ(T(X))\|^2 / \|T(X)\|^2$
- A primary observable is actually the best estimator, from a statistical point of view, which can be chosen.
- This is the starting point for a precise definition of the wave function, that is a function : $W: M \times F \to \mathbb{R}$ such that $W(m,y) = \Pr(\Phi J(X)(m) = y | X))$ is the probability that the measure of the value of any primary observable $\Phi J(X)$ at m is y.

# CHANGE OF VARIABLE

- Theorem 1 : The same system is represented by the variables $X=(X_1,...XN)$ and $X'=(X_1',...XN')$ which belong to open subsets O,O′ of the infinite dimensional, separable, Fréchet vector space V. There is a continuous map $U:V \rightarrow V$, bijective on (O;O′), such that X and $X'=U(X)$ represent the same state of the system. For any observable Φ of X, and Φ′ of X′ : $\Phi' \circ U = U \circ \Phi$. Moreover U preserves the positive kernel on V (the positive kernel plays a role similar to the probability of transition between states of the Wigner's Theorem). Then :
- 1) there is a unitary, linear, bijective map $\mathbf{U} \in \mathcal{L}(H;H)$ such that : $\forall X \in O : \mathbf{U}(T(X)) = T(\mathbf{U}(X))$ where H is the Hilbert space and T is the linear map : $T:V \rightarrow H$ associated to X,X′
- 2) U is necessarily a bijective linear map.
- 3) For any observable Φ, Φ′: $W'=\Phi'(V)$ is a finite dimensional vector subspace of V, isomorphic to $W=\Phi(V):W'=U(W)$
- 4) the associated operators $\mathbf{\Phi} = T \circ \Phi \circ T^{-1}$, $\mathbf{\Phi}' = T \circ \Phi' \circ T^{-1}$ are such that : $\mathbf{\Phi}' = U \circ \mathbf{\Phi} \circ U^{-1}$ and $\mathbf{\Phi}'(H)$ is a vector subspace of H isomorphic to $\mathbf{\Phi}(H)$

- One of the consequences of this theorem is that there should exist a unified system of units.

# CHANGE OF VARIABLES THROUGH A GROUP

- Theorem 2 : The system is represented by fixed variables, and the measures are taken according to procedures which change with $g \in G$ : $(V,U)$ is a representation of the group G : $U(g) \in L(V;V)$

then:

- 1) $(H,\mathbf{U})$ is a unitary representation of the group G with $\mathbf{U}(g)=T \circ U(g) \circ T^{-1}$. If G is a Lie group and U is continuous then U is smooth and $(H,\mathbf{U}'(1))$ is an anti-symmetric representation of the Lie algebra $T_1G$
- 2) For any observable $\Phi \in L(V;W)$: $(W,U)$ is a finite dimensional representation of G, and $(H,\mathbf{U})$ a finite dimensional unitary representation of the group G

# CHANGE OF VARIABLES THROUGH A GROUP

- This last result is especially important in Physics. Any unitary representation of a compact or finite group is reducible in the sum of orthogonal, finite dimensional, irreducible unitary representations. As a consequence the space V of the variables X has the same structure. If, as it can be assumed, the state of the system stays in the same irreducible representation, it can belong only to some specific finite dimensional spaces, defined through the representation or an equivalent representation of G. X depends only on a finite number of parameters. This is the starting point of quantization.
- If G is commutative we have representations on spaces of functions through Fourier transform.

# EVOLUTION OF A SYSTEM

- V is an infinite dimensional separable Fréchet space of maps : $X=(X_k, k=1,N\}::R\to E$ where R is an open subset of $\mathbb{R}$ and E a normed vector space
- 1st Case : General result.
- There is a Hilbert space F, an evolution operator : $\Theta:R\to\mathcal{L}(F;F)$ such that $\Theta(t)$ is unitary, $\forall t:X(t)=\Theta(t)(X(0))\in F$, for each value of t an isometry : $E(t)\in\mathcal{L}(H;F)$ such that $E(t)T(X)=X(t)$
- 2nd Case : Pseudo-determinist evolution
- If, for any fixed $\theta\in\mathbb{R}$ the variables $X'(t)=X(t+\theta)$ and $X(t)$ represent the same state of the system, and the evaluation map : $E(t):V\to E::E(t)X=X(t)$ is continuous, then :
- 1) there is a continuous map $S\in\mathcal{L}(V;V)$ such that :$E(t)=E(0)\circ\exp tS$

$X(t)=(\exp tS\circ X)(0)$ and the operator $S=T\circ S\circ T^{-1}$ associated to S is anti-Hermitian

- 2) there are a Hilbert space F, a continuous anti-hermitian map $S\in\mathcal{L}(F;F)$ such that $X(t)=(\exp tS)(X(0))$. The maps X are smooth.

# INTERACTING SYSTEMS

- We have 2 systems S1,S2, meeting conditions as previously, represented by similar systems, which interact with each other. We want a representation by a unique system, still accounting for the existence of the interactions, but without their explicit introduction.
- If we want that the model S(1+2) meets a few sensible conditions, the solution is to take the tensor product of the variables specific to $S_1$,$S_2$. Then the Hilbert space of S(1+2) is the tensorial product of the Hilbert spaces associated to each system.

# ENTANGLEMENT AND ITS MISUNDERSTANDINGS

- The key point is the difference between the simple direct product : $V_1 \times V_2$ and the tensorial product $V_1 \otimes V_2$
- A tensor is not necessarily the tensorial product of vectors (if it is so it is said to be decomposable), it is the sum of such tensors. There is no canonical map : $V_1 \otimes V_2 \rightarrow V_1 \times V_2$. So there is no simple and unique way to associate two vectors $(X_1, X_2)$ to one tensor S.
- The simple fact that we consider interactions means that the measure of the state of one of the system shall account for the conditions in which the measure is done, so it shall precise the value of the state of the other system and of the interactions $Z_1, Z_2$.

# CONCLUSION

- Whenever the variables representing a system meet some mathematical conditions one retrieves all the features pictured in the axioms of Quantum Physics. So these features do not come from some bizarre properties of the World, but from the way we represent it in our models.
- All the previous results are mathematical, proven theorems. They can be implemented safely, in precise conditions. They provide a safe way in Quantum Physics, beyond what is usually seen as “usual way of computation”.
- They provide guidelines in the modelling of systems in any domain (for instance in Structural Engineering).
- But these features of the models that we can design are them only option that we could choose to follow, or not ?

# COMMON MODELS IN THEORETICAL PHYSICS

- General theories in Physics, aiming at representing a significant domain of the natural world, are built more or less according to the same lines, be it Newton's Mechanics or Einstein's theory of gravitation. They acknowledge objects in Nature, with assumed properties (forces, momentum, energy…), which transform according to First Principles. All this is represented in Mathematical models which are then the base to explain experiments and check the theories. So they proceed according to assumptions which are not those of Quantum Physics, but also according to a different path.
- I give the example of such a theory, which is mine and covers all the known properties, from gravitation to nuclear forces.

# THE OBJECTS OF THE THEORY

- **The Universe** : this is the universal container in which everything stays
- **Material bodies** : matter in all its form, from elementary particles to galaxies. Their main properties : they occupy a precise location in the universe at any time, they have a shape (they can be rotated in space).
- **Force fields** : they have 3 properties :

- they exist everywhere;
- they interact with material bodies, their value change as well as the properties of material bodies, notably their motion
- they propagate : their value change from one point to another in the vacuum, without interacting with material bodies.

- **The observer** : he has free will. He can choose the system, its components, the time and location of the experiment, the units, and from where he observes and measures.

# THE FIRST PRINCIPLES

- Principle of causality : “There is no effect without a cause”. An event A can be the cause of an event B, B can be the cause of A, or A and B can be not related. Physic is determinist, but the order of the events does not depend on the observer.
- Principle of Relativity : “Physical laws do not depend on the observer”.
- Principle of Locality : ”The outcome of any physical process occurring at a location depends only on the values of the involved physical quantities at this location”.
- Principle of Conservation of Energy : “In any process the balance of energy exchanged between the objects of a system is preserved".
- The Principle of Least Action : is a generalization of the Principle of conservation of momentum, for a complex system, in any process.

# THE GEOMETRY OF THE UNIVERSE

- From the existence of charts, the Universe can be represented by a 4 dimensional real manifold. Out of Cosmology, which involves some basic logical assumptions (where is the observer ?), we consider a relatively compact area.
- The 4 dimensions are not equivalent : time is not measured as space, one cannot travel in time, and the Principle of causality implies the existence of a metric with the signature (3,1) or (1,3).
- As a consequence, to each observer is assigned, at any point, an orthonormal frame, called a tetrad, with respect to which the motion (transversal and rotational) of material bodies is measured. Moreover it defines a 4 dimensional density, as well as a 3 dimensional spatial density.
- The standard observer is assumed to be spatially immobile, and it defines a standard chart, as well as a set of tetrads which are used for reference. There are future oriented and past oriented vectors.

# CLIFFORD ALGEBRAS

- On a n dimensional real or complex vector space F endowed with a symmetric bilinear form the Clifford algebra Cl(F, ρ) is a $2^n$ dimensional algebra on the same field, built by adding the internal product of vectors u,v with the property : $u.v + v.u = 2\rho(u,v)$. The algebra contains the scalars, the vectors of F and the sum of ordered products of vectors. The exponential is well defined, transposition is an involution which reverses the order in the product of vectors, there is a scalar product.
- The transformations which preserve the scalar product in F are given by the adjoint map Adg, defined by : $\mathrm{Adg}Z = g\cdot Z\cdot g^{-1}$.
- With the tetrad defined by an observer one can define a Clifford bundle Pg[Cl(3,1),Ad] with Clifford vectors located at each point.

# MOTION OF MATERIAL BODIES

- Material bodies have trajectories p, there is a unique parameter $\tau$, the proper time, such that it is the representation of a given path $p(\tau)$.
- Observers follow trajectories with respect to their proper time t. The velocity of a material body is $V=dq/dt=c\varepsilon_0+v$ where v is the spatial speed. The standard observer is spatially immobile and for him V has a fixed Lorentz metric $-c^2$.
- From which one deduces the relation between the proper time of a material body and the time t at which a standard observer locates the body.
- Because it is assumed that there is a tetrad attached to a material body, there is also a rotation.
- The right representation of the motion of a material body is then by an element of the group Spin(3,1):$\sigma=\exp Tr\cdot\exp Tw$ which is the combination of a translation Tw and a rotation Tr, defined by 6 real parameters which can be computed from the usual quantities.

# MECHANICS

- The First Law of Newtonian Mechanics is the law of inertia : the momentum, defined as $P = mv$, is invariant without an external intervention, which can be by contact or action at a distance. From there one has the kinetic energy on one hand, and, painstakingly, the torque and rotational momentum.
- A simplistic extension to the relativist picture defines a translational relativist momentum by $P=mU$ with the proper velocity $U=dq/d\tau$. But, because $K=\langle P,P\rangle/m=-mc^2$ there is no simple definition of the kinetic energy, and the rotational motion is ignored.
- With a purely QM starting point, Dirac proposed the equation $i\partial\psi)/\partial t=-i\sum\gamma\alpha(\partial\psi/\partial\xi\alpha)+m\gamma_0\psi$ with 4 complex matrices $\gamma i.\gamma j+\gamma j.\gamma i=2\eta ijI_4$. The variable $\psi$, called spinor, replaces the momentum, and belongs to a vector space which is the representation of the Clifford algebra. It accounts for the antiparticles and is the basis for the Standard Model.

# COMPLEX AND REAL CLIFFORD ALGEBRAS

- There is a morphism C:Cl(3,1)→Cl(ℂ,4) between the real and complex Clifford algebras. With it one can define a real structure on Cl(ℂ,4): one element Z of Cl(ℂ,4) can be written : Z=X+iY where X,Y belongs to Cl(3,1), however in the tetrad they are then expressed by complex components. One can then define complex conjugation and hermitian scalar product.
- The Clifford algebras Cl(3,1) and Cl(1,3) are not isomorphic and do not have the same matricial representations. However, the representation of Cl(ℂ,4) provides a real matricial representation of both Cl(3,1),Cl(1,3).
- What is important in the Dirac's equation, this is the complexification. The matricial representation can be useful but it is not necessary. And we can easily introduce the equivalent of the Dirac's spinor in the geometry of General Relativity.

# SPINOR IN GENERAL RELATIVITY

- The inertial characteristics of a particle are represented by a fixed vector $\psi_0 \in Cl(\mathbb{C},4)$. They are measured by an observer as a vector, representing the kinematic state of the particle $\psi(t)=Ad\{C(s(t))\}\psi_0$ of the associated vector bundle $PC=P[Cl(\mathbb{C},4),C(Ad)]$, where $s(t) \in Spin(3,1)$ represents the geometric state of the particle for the observer.
- The momentum is represented in the 1st jet extension $J^1PC$ of PC. In a continuous motion : $\delta\psi = d\psi/dt$ and belongs to the Lie algebra $T_1C(Spin(3,1))$. It can easily be extended to deformable solids and to study geometric symmetries of solids.
- The variation of kinetic energy is then given by the Hermitian product $\delta K=(1/i)\langle\psi_0,[\delta C(s),\psi_0]\rangle$ with $\delta C(s)=C(s)^{-1}\cdot(dC(s))/dt)$.

# GAUGE FIELD THEORY

- The idea of action at a distance is replaced by the interaction of particles with force fields. Its most achieved representation is the gauge field theory, which is general.
- The state of a particle is represented by a vector $\psi$ of an associated vector bundle $P[F,\vartheta]$ with the group U. It changes on the trajectory under the action $\vartheta$ of the force field, which is represented by a connection. Its action on a section of the vector bundle is represented by a covariant derivative $\nabla\psi$.
- Because $\langle\psi(\tau),\psi(\tau)\rangle=\langle\psi_0,\psi_0\rangle$ we can take as lagrangian $L=(1/i)\langle\psi,\nabla\psi\rangle=\sum TaQa$ where the charges Qa are real scalars and Ta are vectors of a basis of the Lie algebra $T_1U$.
- The force field is then associated to a Lie group U.

# PROPAGATION OF FORCE FIELDS IN THE VACUUM

- Force fields propagate at the speed of light c : for any observer the 3 dimensional hypersurfaces of simultaneous events are the set of points where things happen.
- In a gauge field theory the propagation is represented by a tensor : the 2 form $\mathcal{F}\in\Lambda_2(M;T_1U)$ valued in the Lie algebra of the group U.
- In the vacuum propagation occurs along preferred curves at a speed such that the points which are reached after some delay belong to a 2 dimensional hypersurface : a wave.
- The First Principle of Optics says that "light propagates in straight lines". In Einstein's theory they propagate along geodesics. My assumption is that they propagate along Killing curves. They constitute a Lie algebra of dimension at most ten on a 4 dimensional manifold, and represent the symmetries of the metric, which is the physical part of the Geometry

# ISOMETRIES

- Isometries are a subgroup of diffeomorphisms, they preserve the metric, and its Lie algebra are the Killing vector fields, V on TM, such that their flow f(m)=Φ{V}(τ,m) is an isometry. The condition is that the Lie derivative £ along the vector V is null : £Vg=0.
- An isometry f can be extended to a Clifford bundle isomorphism (f,F) on PC. For a propagation curve this is necessarily a spatial rotation with the expression F(m)(Z)=AdC(S(m))(Z) where S is a spatial vector defined at each point m.
- There is a unique propagation curve going through 2 points, or going through a point with a (time) given vector.

# GRAVITATION AND THE EM FIELD

- In the Newton's theory of gravitation the charges are equal to the mass, this is experimentally checked.
- The Einstein's gravitation theory, which is different from General Relativity, actually replaces gravitation by the action of inertia in a curved space time.
- But we can easily adjust the gauge field theory to incorporate gravitation in the General relativity geometry : the group is the spin group Spin(3,1) and the charges are represented by the spinor vector.
- Similarly the electromagnetic field (EM) is represented by the group U(1) acting on spinors with the complex structure. There are 3 possible, irreducible, non equivalent representations of U(1), which are unidimensional and associated to a charge equal to +1,-1, or 0.

# NUCLEAR FORCES

- Nuclear forces encompass weak and strong interactions, associated to the groups SU(2) and SU(3).
- Over all we have 24 elementary particles, and 24 antiparticles :

- 3 generations of pairs of leptons : e,ν; leptons of the same flavor have the same electroweak charge ;
- 3 generations of 6 quarks, differentiated by the flavor (u,d) and the color (r,g,b). Quarks of the same flavor and generation have the same mass; quarks of the same flavor have the same electroweak charge.
- Each particle has an associated anti-particle with the same mass but different charges

. These features impose conditions on the structure of the charges, and then on the group U

# THE UNIFIED MODEL

- The group U is the 16 dimensional real Lie group $\{u \in Cl(\mathbb{C},4)::CC(u^{\wedge})\cdot u=1\}$. The definition holds for both signatures.
- U is a unitary group and $(Cl(\mathbb{C},4),\vartheta)$ is a unitary representation of U.
- Its Lie algebra is real but with a basis which is different from the real basis of C(Cl(3,1)).
- The standard action is $\vartheta:U\rightarrow\mathcal{L}(Cl(\mathbb{C},4);Cl(\mathbb{C},4))::\vartheta(u)(\psi_o)=\exp(iA)Adu(\psi_o)$
- Antiparticles are defined in the contragredient representation, with the same group and vector space, but the action is :

$\Theta':U\rightarrow\mathcal{L}(Cl(\mathbb{C},4);Cl(\mathbb{C},4))::\vartheta'(u)(\psi_o)= \exp(-iA)AdCC(u)(\psi_o)$

They have the same gravitational charges, opposite EM charge, and identical or opposite charges for the other fields.

# SYSTEM OF PARTICLES

- With this representation it is possible to write the equations for the equilibrium of a system of particles and their field, using the Principle of Least Action, variational derivatives and the propagation along Killing curves.
- Particles are sources of the field, and the current related to the field propagates from an interaction with a particle in spherical waves. A particle is seen only once along a definite spatial direction, the action of the field decreases roughly as the square of the distance to the particle. The metric is defined by the field.

# BOSONS

- Continuous process are the rule but discontinuous process can occur in collisions or to assure another equilibrium.
- The connection is always continuous, but not necessarily continuously differentiable. If there is a discontinuity of the derivatives there can be a right and a left derivative which are not equal. The discontinuity, computed as a difference, is a one form which propagates along curves. It is then similar to the motion of a particle and leads to the representation of bosons by $B(\tau)=[AdC(S(\tau))]\Delta A(Y)\in T_1U$ such that its Hermitian scalar product $\langle B,B\rangle=\langle \Delta A(Y),\Delta A(Y)\rangle$ is constant along the propagation curve.
- Bosons interact with particles, the trajectory is still continuous, but no longer continuously differentiable.
- In this picture the "mass" of a boson corresponds to the 6 components of B associated to $T_1Spin(3,1)$

# STABLE SYSTEMS

- A system of N particles in the EM field is usually not stable, as well as N planets in their gravitational field. The only known stable free hadron, composite particles composed of quarks and antiquarks, is the proton with 3 quarks. However stable systems are ubiquitous : nuclei are extremely resilient, as well as atoms.
- One can show that a system of N particles can be stable if their location follow precise symmetries (similar to a crystal). Then the system behaves as a single composite particle with a specific charge and their state is quantized.
- Systems of composite particles can then be studied by accounting only for the relevant charges and fields.
- Such stable systems follow the mathematical prescriptions of QM, if the system is confined to a limited area.

# QUESTIONS

- Stable systems, characterized by a highly geometric, symmetric, organization, are dominant in Nature. However they do not appear as the consequence of the action of the objects in Nature.
- Antiparticles exist and are stable. However the world which is accessible to us seems to be composed uniquely of particles.
- The physical world seems to follow very specific requirements, according to our Mathematics, be it in QM or Unitary Models. Does the Universe comply to the Mathematics, or does Mathematics comply with the Universe ? What is the true meaning of our mathematical proofs ?

# A STRANGE QUESTION

- In “Research Gate”, a very serious scientific internet site, appeared  some months ago a question, which has aroused long and passionate discussions : “**Do irrational numbers exist in Nature** ?”. After all the proof of a scientific truth is the production of a material evidence. And, materially, it is impossible to produce an irrational number, which has an infinite number of digits.
- To answer this question we have to go back to the revolution which happened in Mathematics at the beginning of the XX° century. It faced a crisis, which has been solved by the distinction between a “motor”, the engine which enables to make proofs (mathematical logic) starting from very basic propositions (they are true or false), and the formal systems, which are arbitrary collection of definitions, for numbers and sets. Altogether they give back the mathematics as we know. So the existence of irrational numbers are just a consequence of the introduction of numbers.
- And in our Unitary Model, we have added an “object”, which is fundamental in Relativist Physics : the Observer.  Any model, then any “objective” check of a theory, follows the prescriptions of the observer.