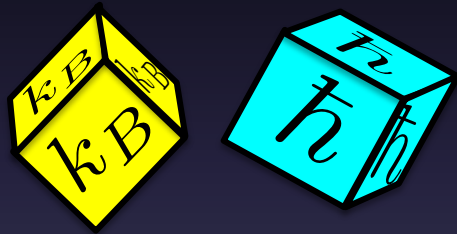


What is quantum in quantum randomness?

Alexia Auffèves & Philippe Grangier

with the support of N. Farouki and the quantum foundations group of Grenoble



- Grangier and Auffèves, Found Phys 46, 121 (2016)
- Auffèves and Grangier, Sci Rep 43365 (2017)
- Grangier and Auffèves, Phil Trans R Soc A (2018)

Ordinary classical randomness



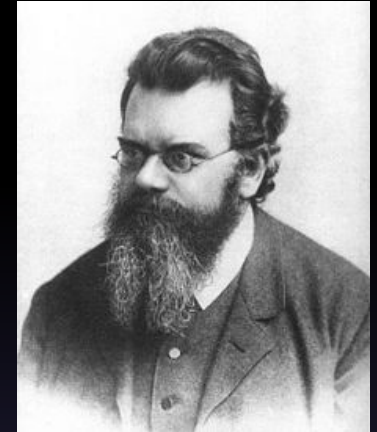
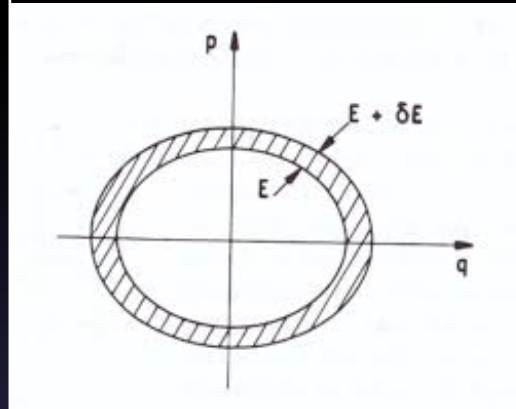
- Classical randomness is due to some ignorance on the « *state of the system* »
- The « real state » is hidden
- It often appears with the lack of information
- It is often subjective



Classical randomness in physics

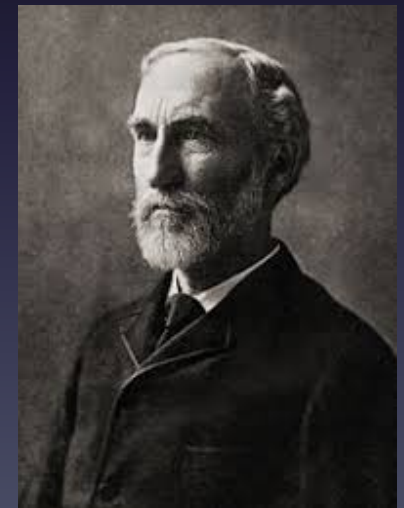


- Statistical physics:
- The « real state » is microscopic/below
- Loss of information while coarse-graining from micro to macro-states
- Randomness and irreversibility are due to the imperfection of our perceptions



Boltzmann

$$S = k. \log W$$



Gibbs

Quantum randomness?



- First interpretation
Irreducible interaction
with the experimental
context (irreducible
coarse-graining)

- The system has a hidden
state, the state is
perturbed
- « *Local realism* » *a la*
Einstein



- Randomness is
bad and
quantum
mechanics is
incomplete

Randomness in a non-local world

Closing the Door on Einstein and Bohr's Quantum Debate

By closing two loopholes at once, three experimental tests of Bell's inequalities remove the last doubts that we should renounce local realism. They also open the door to new quantum information technologies.

by Alain Aspect*

- B. Hensen, et al, and R. Hanson, Nature 526, 682 (2015)
- M. Giustina, et al, and A. Zeilinger, Phys.Rev.Lett. 115, 250401 (2015)
- L. Shalm, et al, and S. Nam, Phys.Rev. Lett. 115, 250402 (2015)

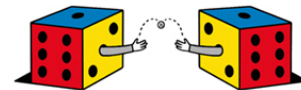
➤ Experimental violations of Bell's inequalities : « The world is non local »

➤ Quantum randomness prevents from communicating faster than light



Nicolas Gisin
L'Impensable Hasard
Non-localité, téléportation
et autres merveilles quantiques

Préface d'Alain Aspect



sciences

Randomness in a contextual world?



➤ « The world is contextual »

➤ How to think quantum randomness in a contextual world?

➤ *Challenge: our intuitions on randomness are non-contextual*



Specker



Kochen

M_1	M_1'	M_2	M_2'	M_3	M_4	M_5	M_5'	M_6
M_3'	M_7	M_4'	M_8	M_7'	M_8'	M_6'	M_9	M_9'
M_{10}	M_{11}	M_{10}'	M_{12}	M_{13}	M_{14}	M_{15}	M_{11}'	M_{13}'
M_{15}'	M_{12}'	M_{16}	M_{17}	M_{14}'	M_{18}	M_{16}'	M_{17}'	M_{18}'
C1	C2	C3	C4	C5	C6	C7	C8	C9

➤ Are there « genuinely quantum » causes of quantum randomness (related to quantization)?

➤ **What is quantum in quantum randomness?**

Outline

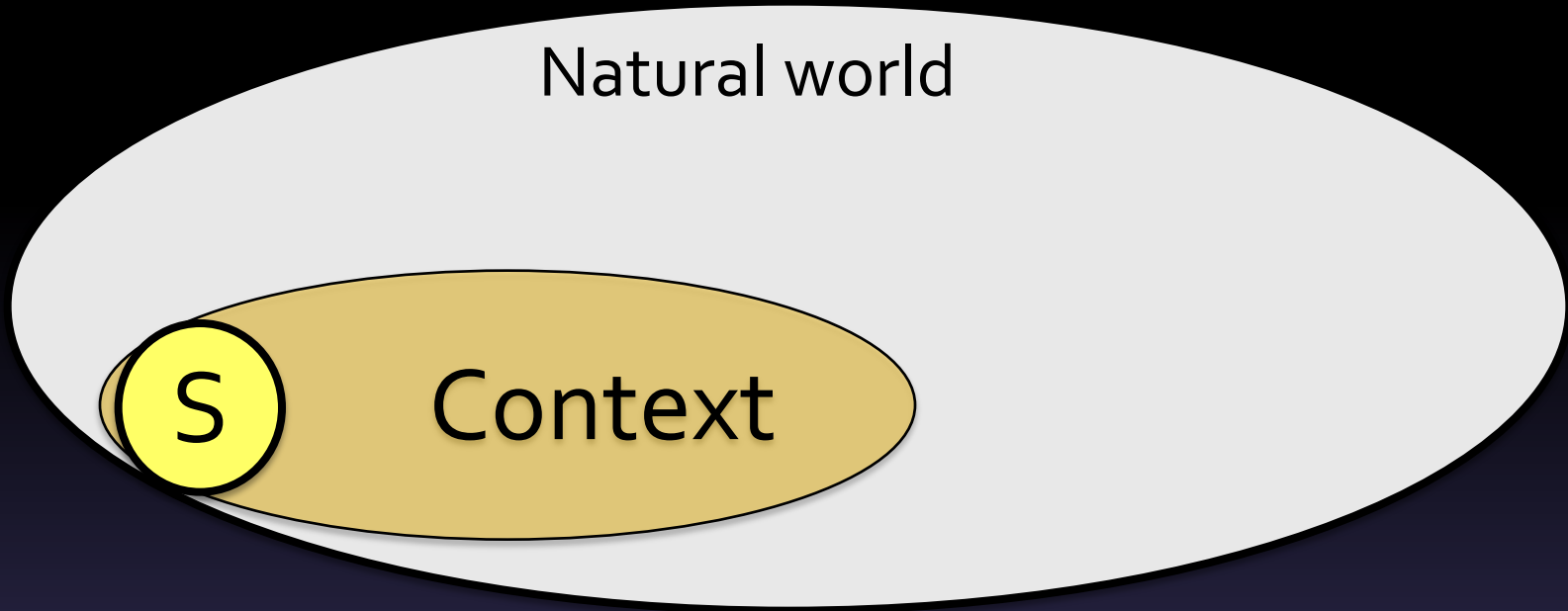
- A contextual approach of reality
- Randomness in a contextual world
- Heuristic derivation of the quantum formalism
- Conclusions & Outlooks

Primitive ontology in physics



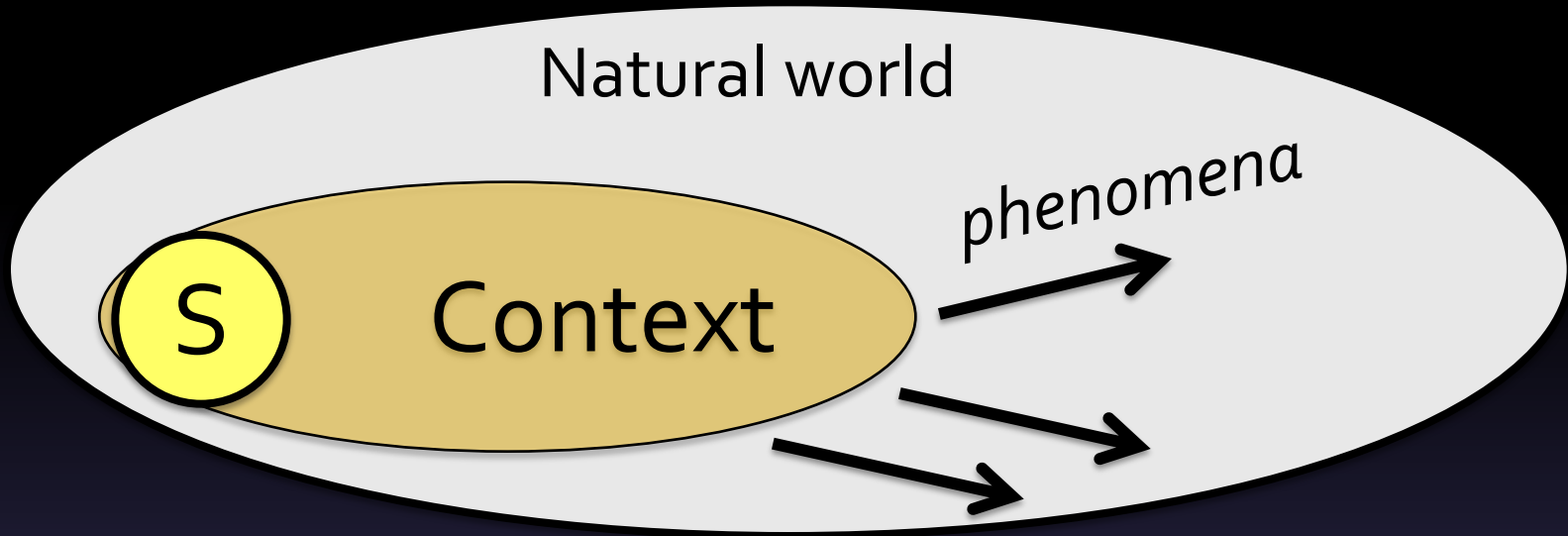
- System = a finite entity of the natural world
- Systems are **objective** = exist if unobserved

Primitive ontology in physics



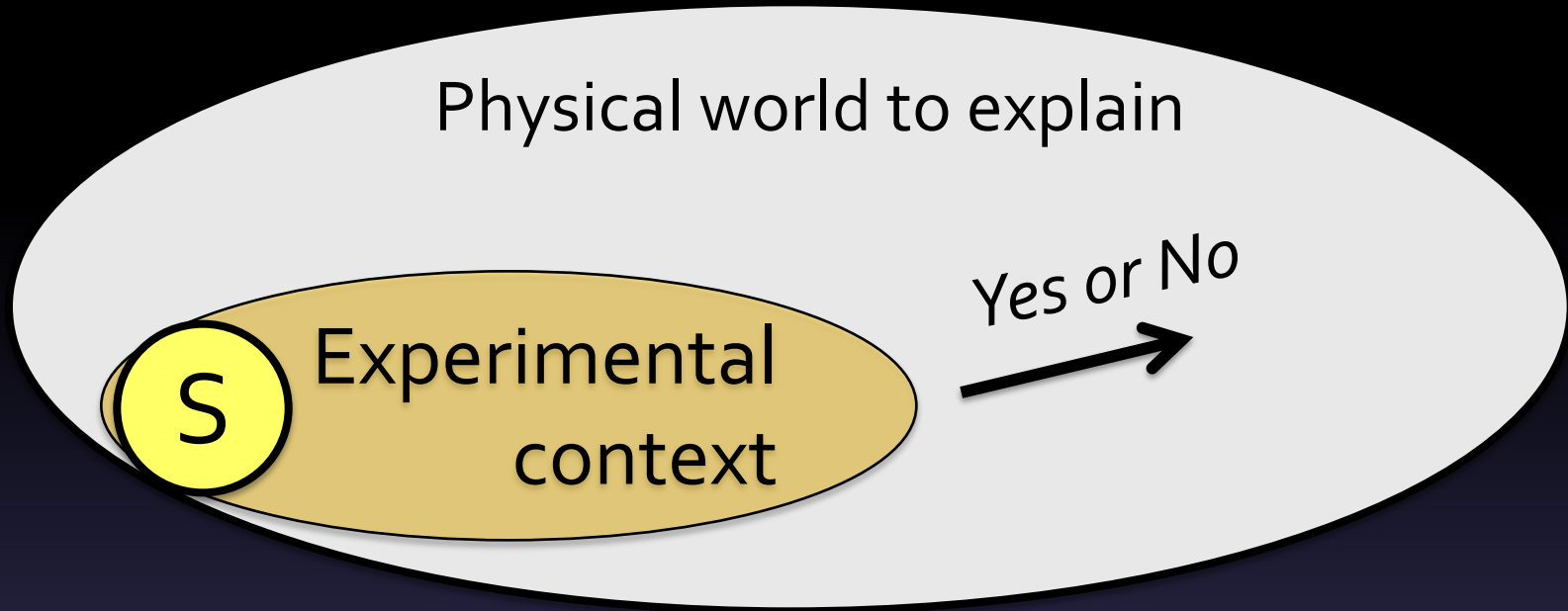
- Context := « around the system »
- Contexts are **objective** = exist even if unobserved
- Contexts are made of the same « stuff » as systems

Primitive ontology in physics



- $\{\text{System} + \text{Context}\} \rightarrow \text{Phenomena} = \text{Events}$ in the natural world
- Phenomena are **objective** = exist even if unobserved
- Phenomena are **actual** = realized

A possible partition

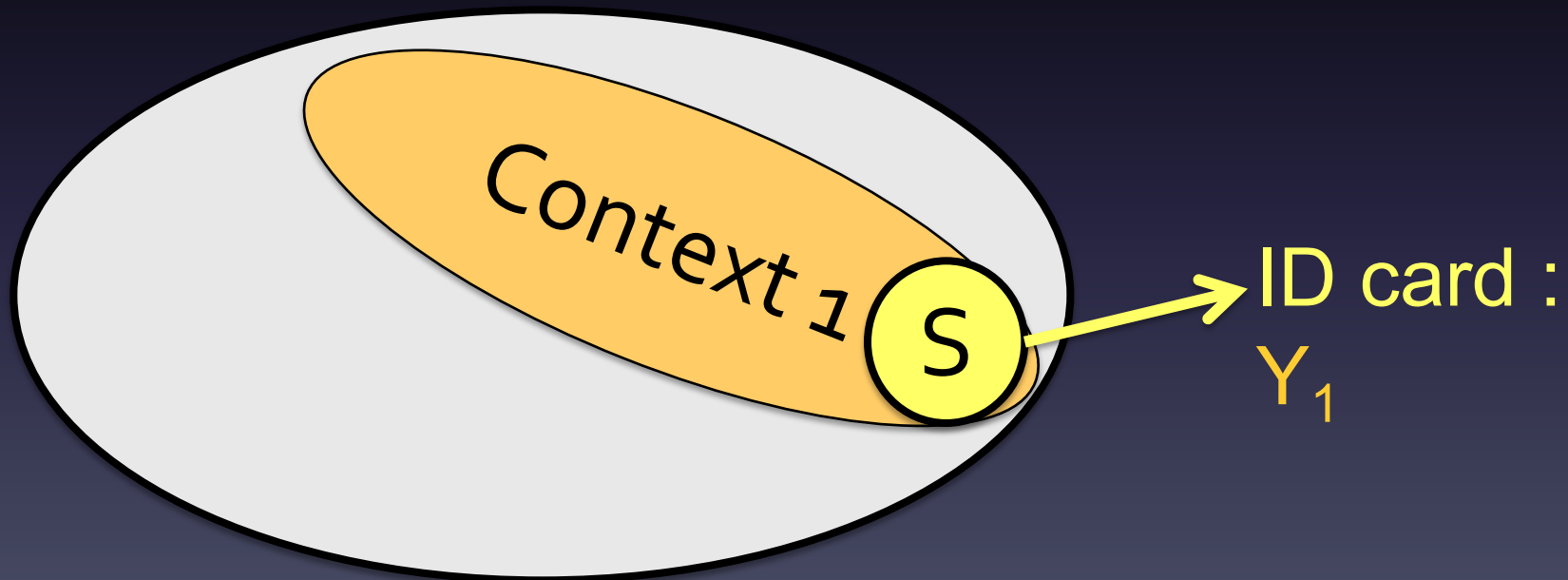


- The context = A measuring device = A question asked to the system
- The phenomenon = A measurement outcome = An answer to the question

What is a state?

Operational approach :

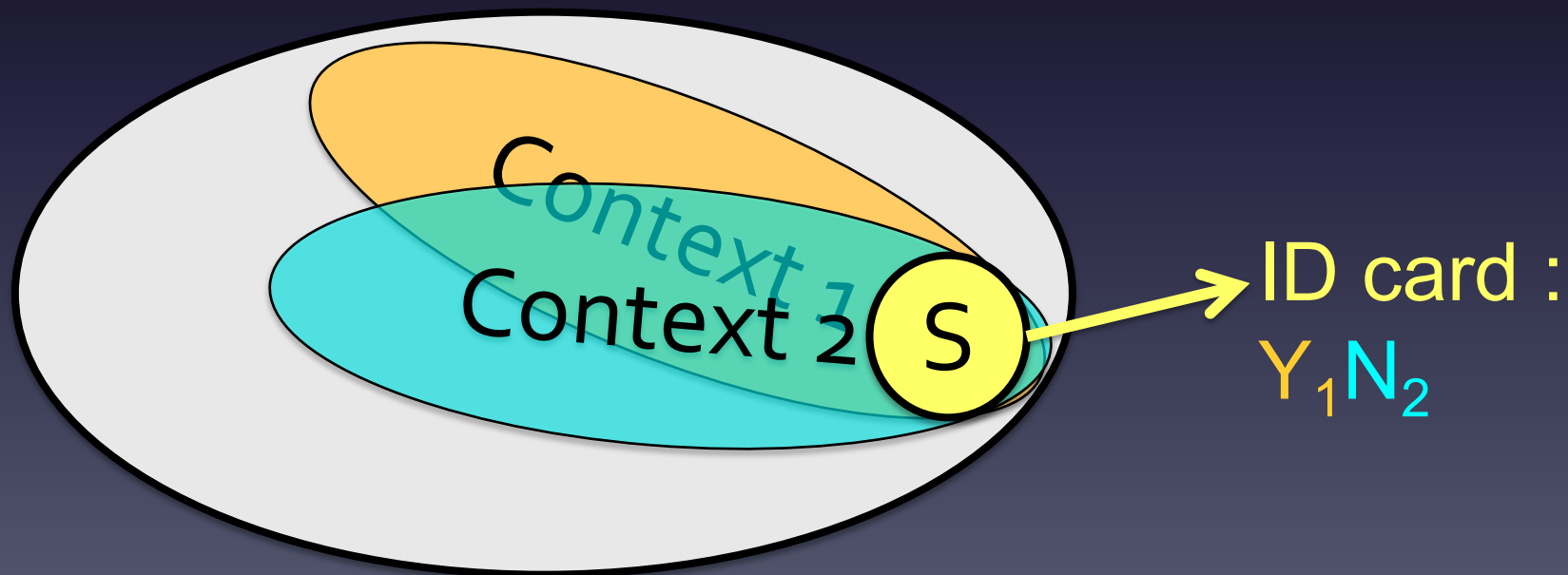
- One builds a state by asking the system a set of questions and filling its ID card
- Each question is asked within a given context
- Each answer is a phenomenon within the given partition



What is a state?

Operational approach :

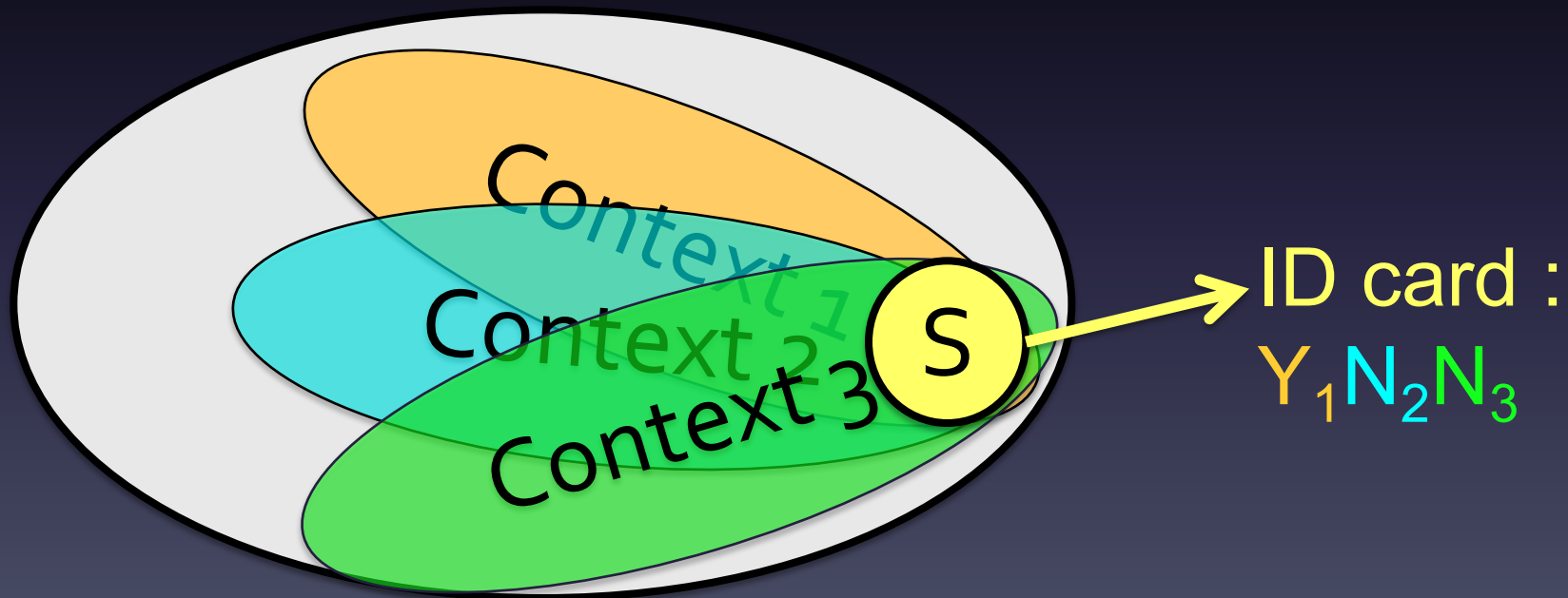
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What is a state?

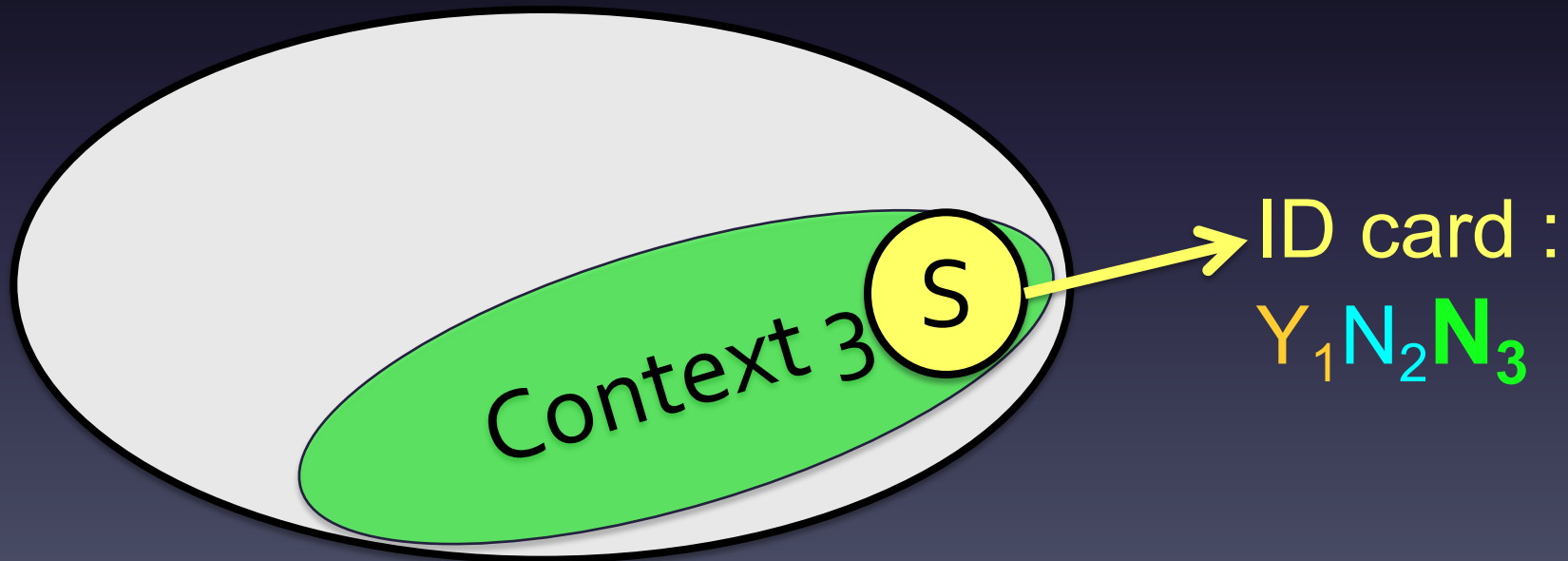
Operational approach :

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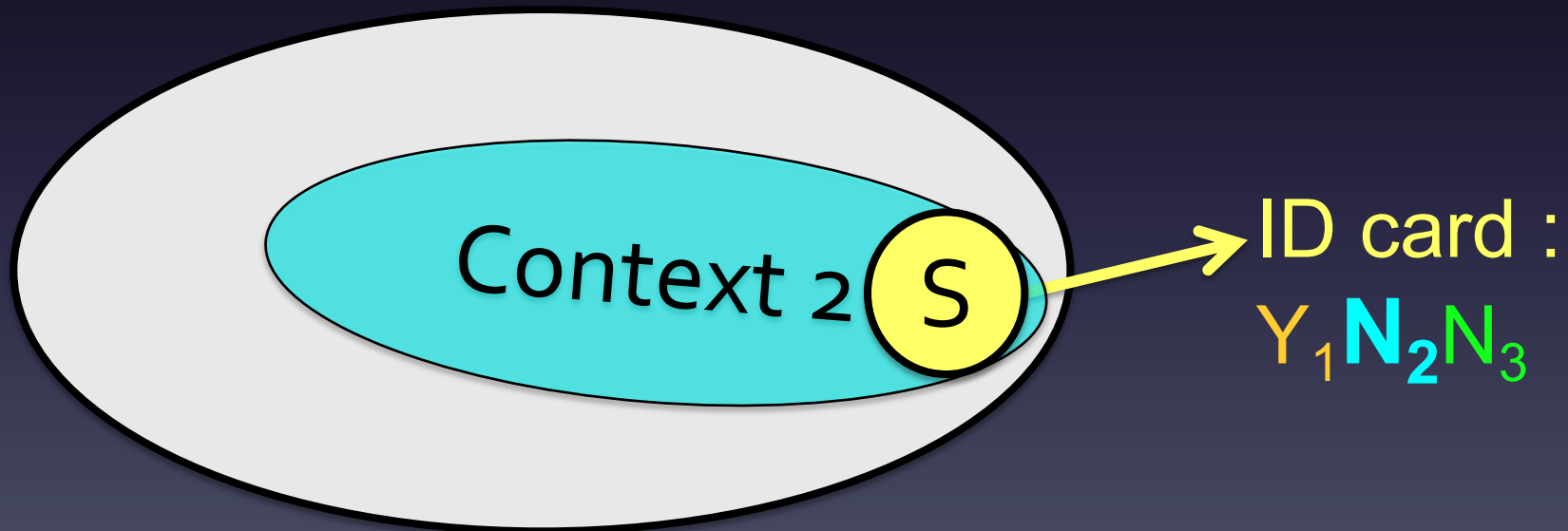
Classical phenomenology

I can obtain repeatably the same answers to the same questions



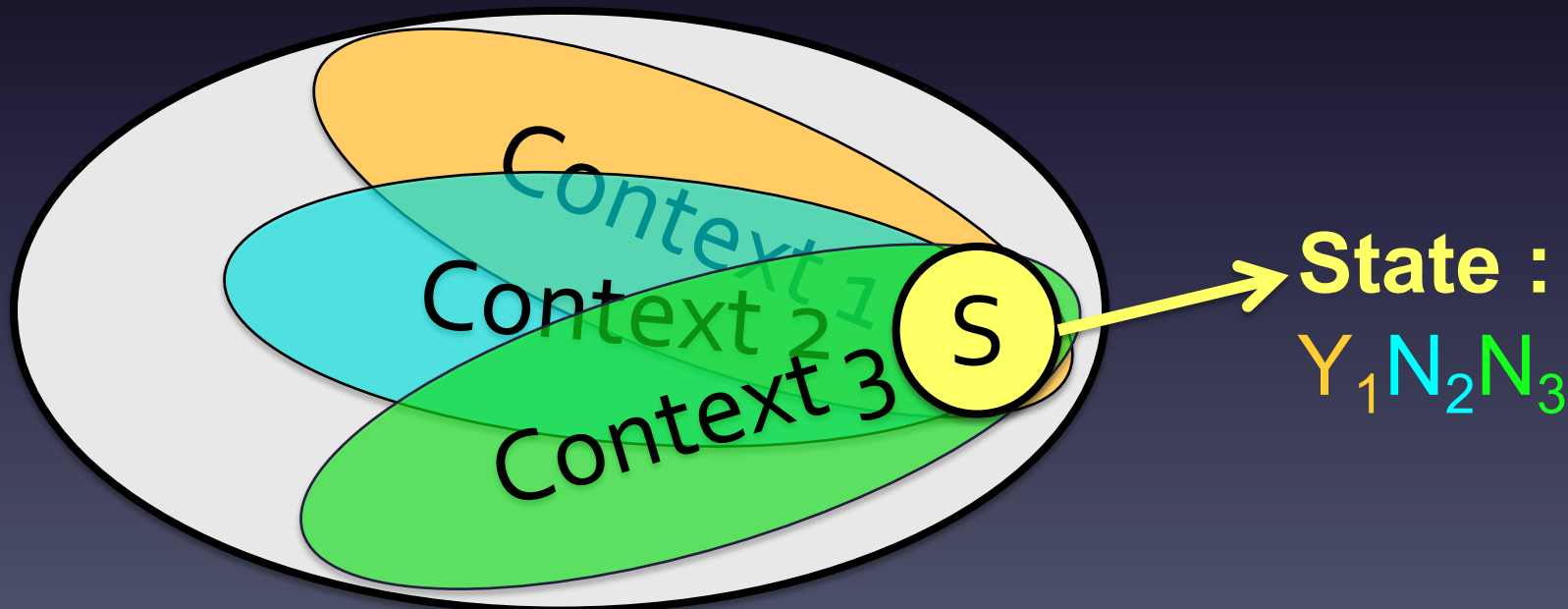
Classical phenomenology

I can obtain repeatably the same answers to the same questions



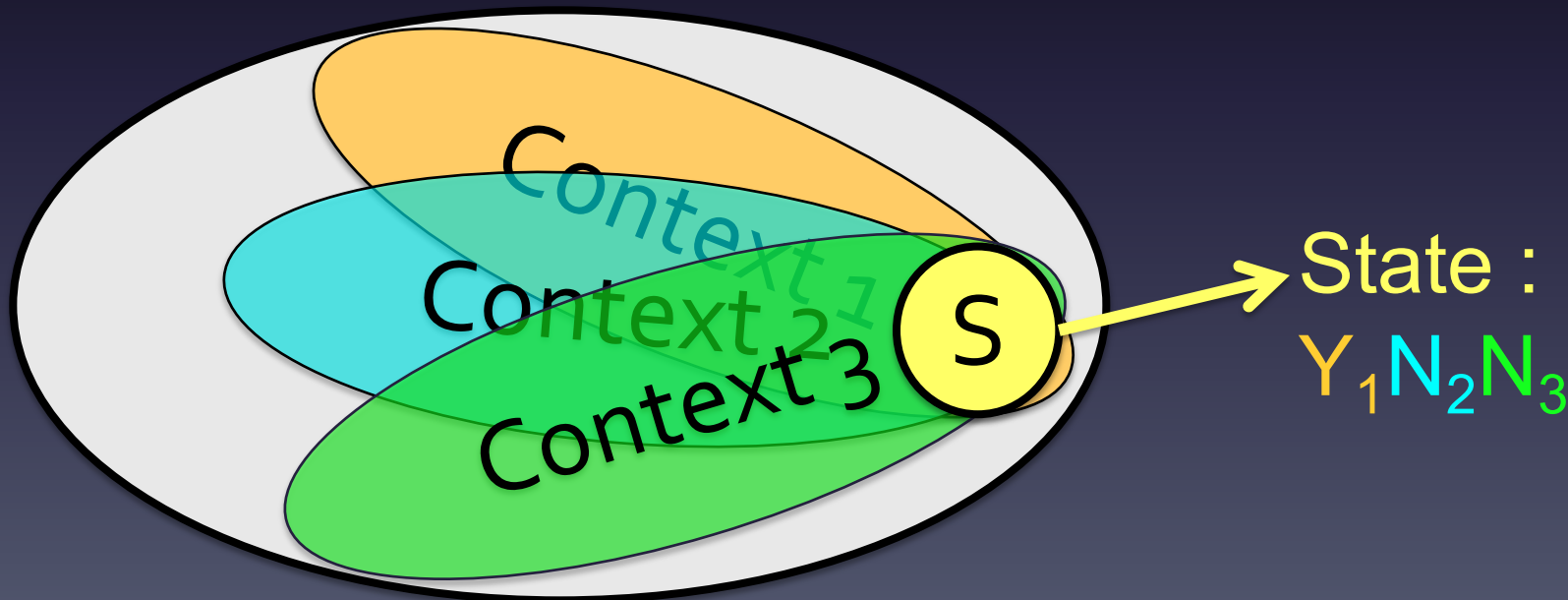
Genealogy of a classical state

- The ID card (*Operational*) -> State (*Ontology*)
- *One explains the repeatable answers by a permanent cause : the existence of a state*
- A state is objective, doesn't need any observer to exist



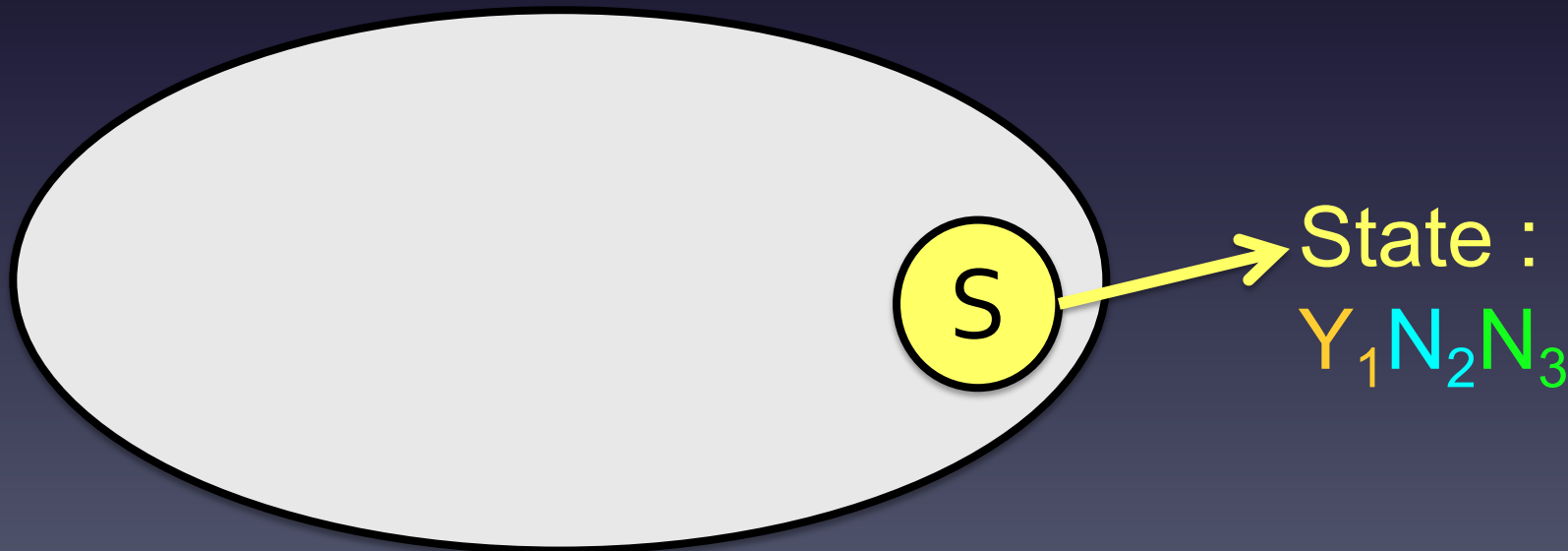
Genealogy of a classical state

- In the classical world, the state does not depend on the ordering of the questions
- I can forget the contexts and attribute the state to the system alone



Genealogy of a classical state

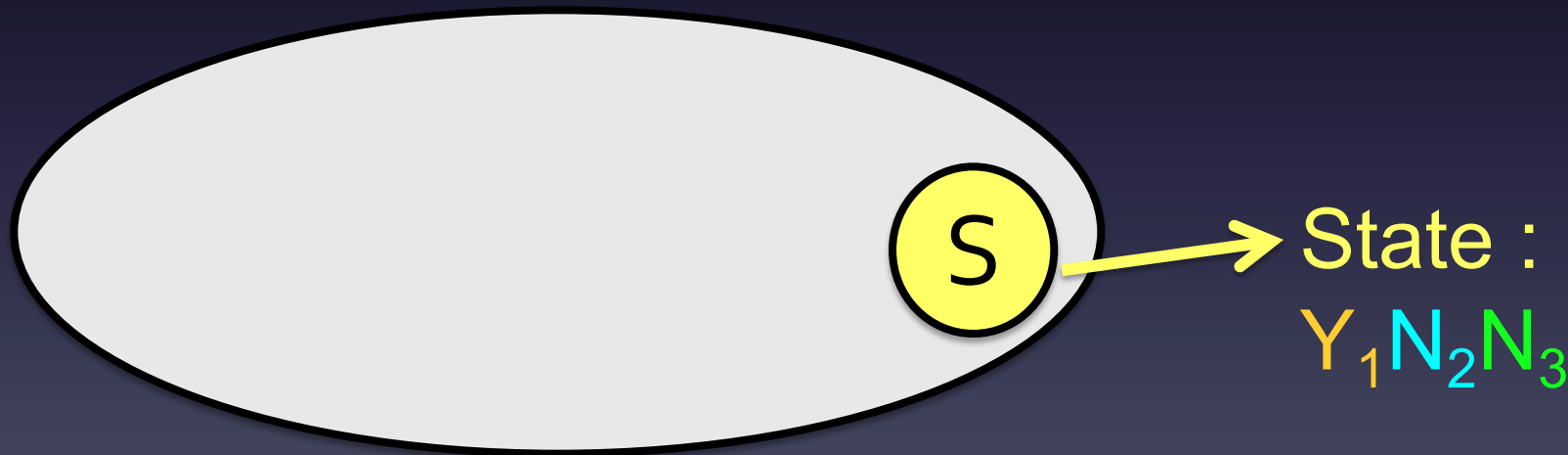
- In the classical world, the state does not depend on the ordering of the questions :
- I can forget the contexts and attribute the state to the system alone
- **Non contextual state**



Genealogy of a classical state

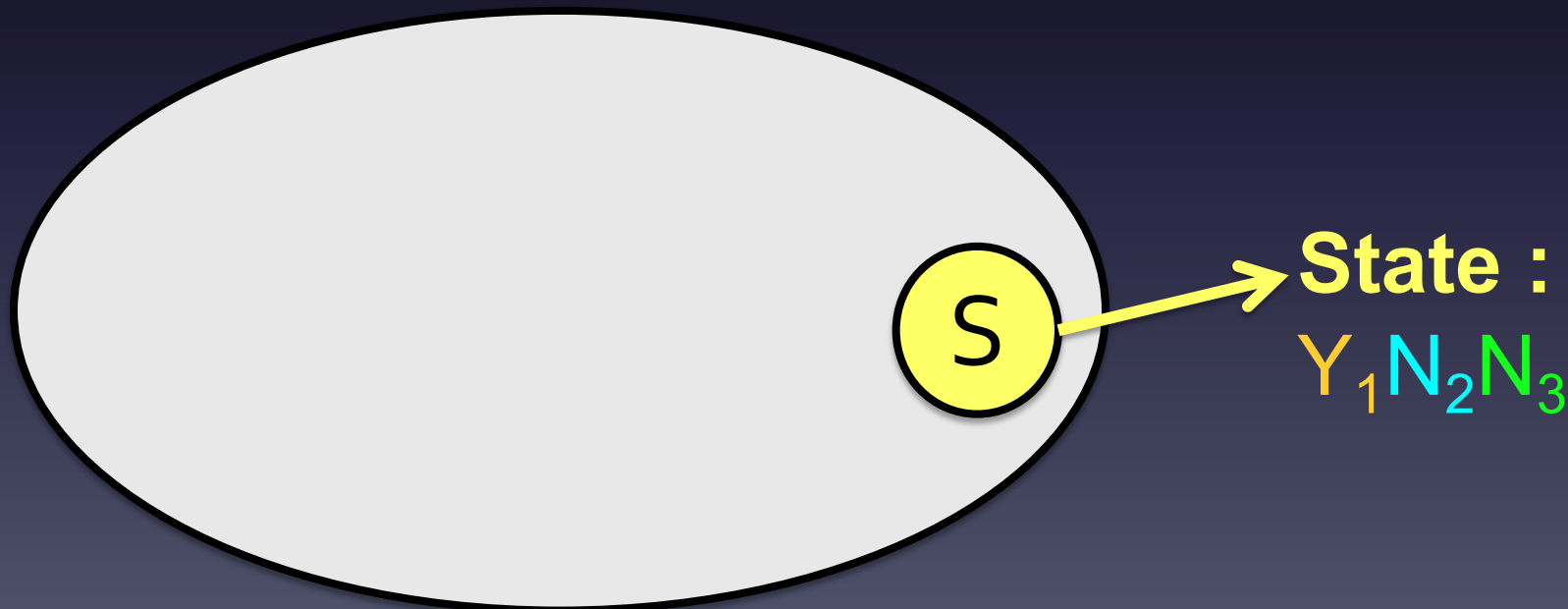
Our natural intuition « States are non contextual » is **built in the classical world**

*Such intuition has deep roots: We perceive classical phenomena continuously, and **for free***



Genealogy of a classical state

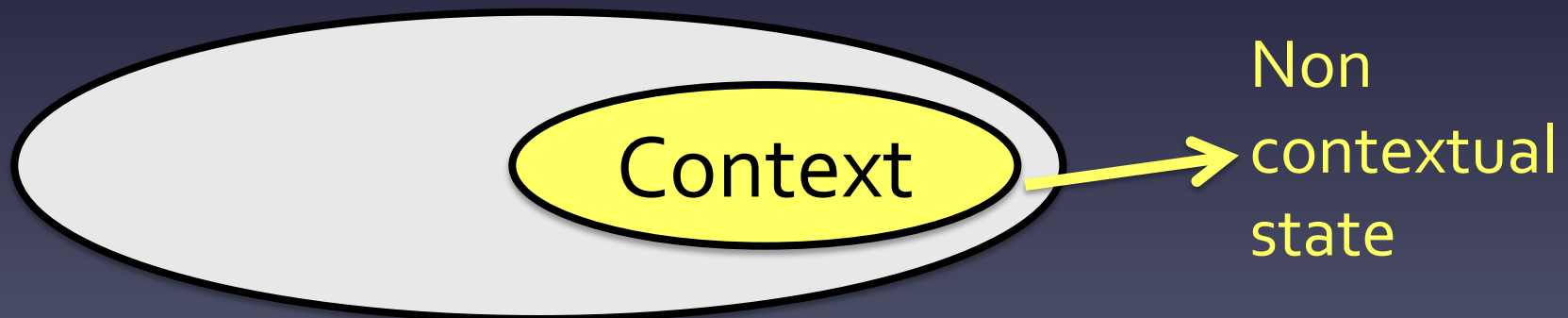
- **Objectivity** => **Inter-subjectivity**
- « The system S is in the state A » is an **information** that can be communicated to/verified by other parties and gives rise to a consensus.



Postulate 0:

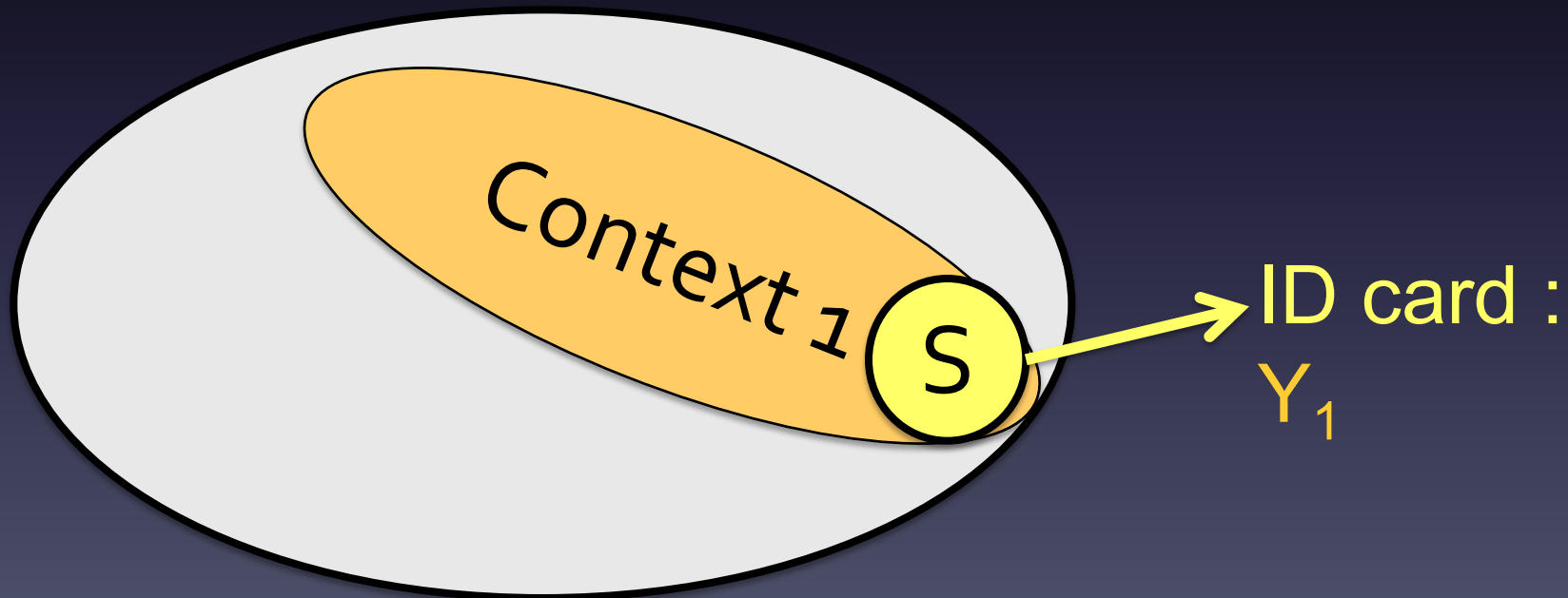
Non-contextuality of contexts

- Contexts have their own states, which do not depend on other contexts (nor on the system they may contain)
- Provides a fixed point to extend ontology beyond classical intuitions



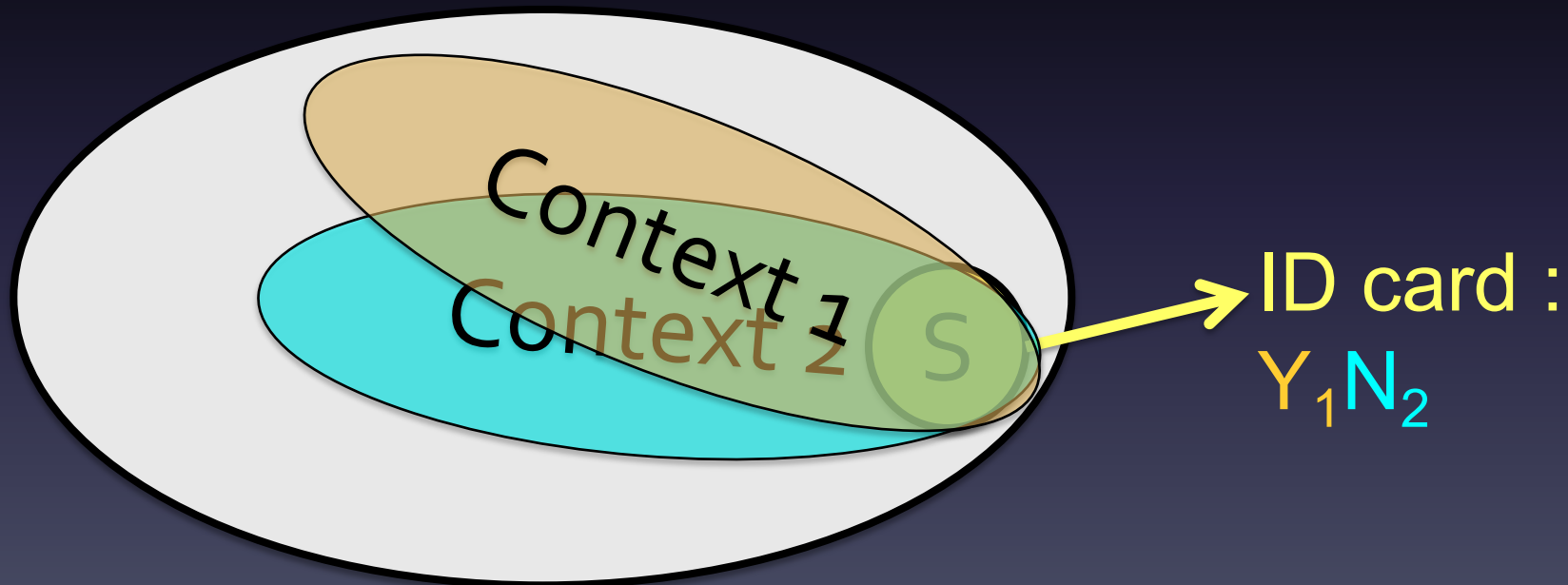
Quantum phenomenology

- Consider a system and a context
- The state of the context can be changed: $C_1 \rightarrow C_2$
- In general the ID card = the answers depend on the ordering of the questions



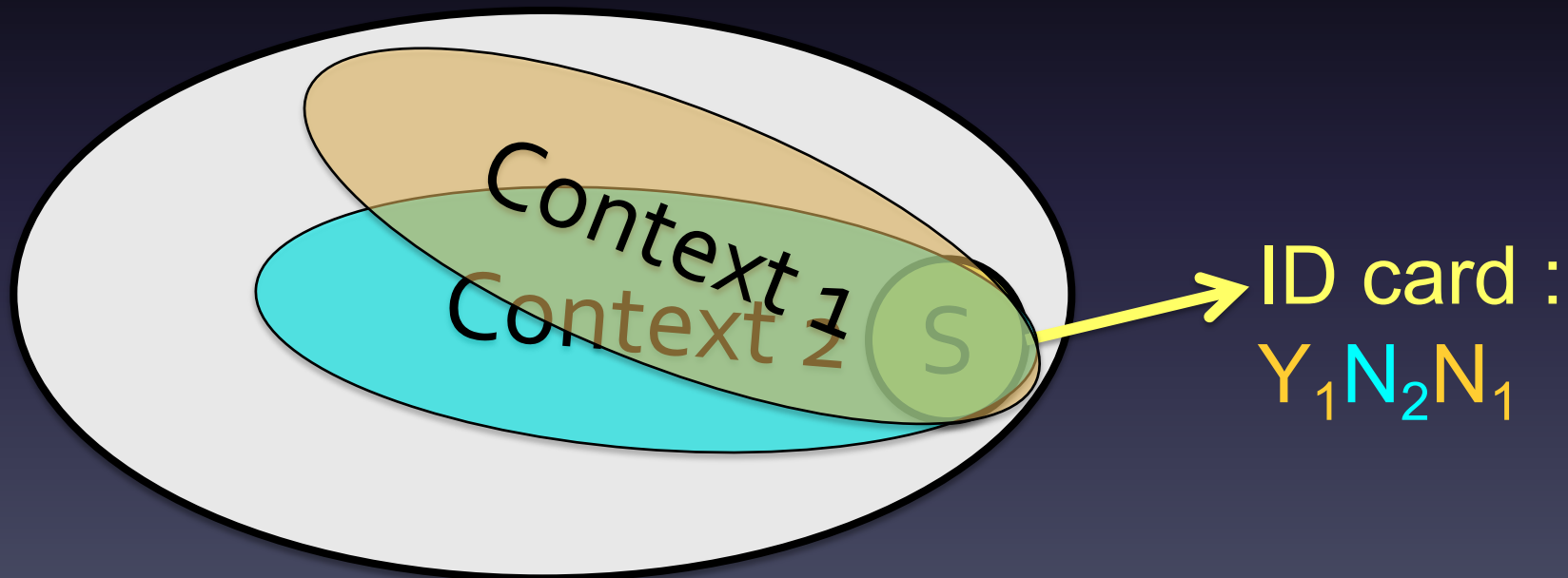
Quantum phenomenology

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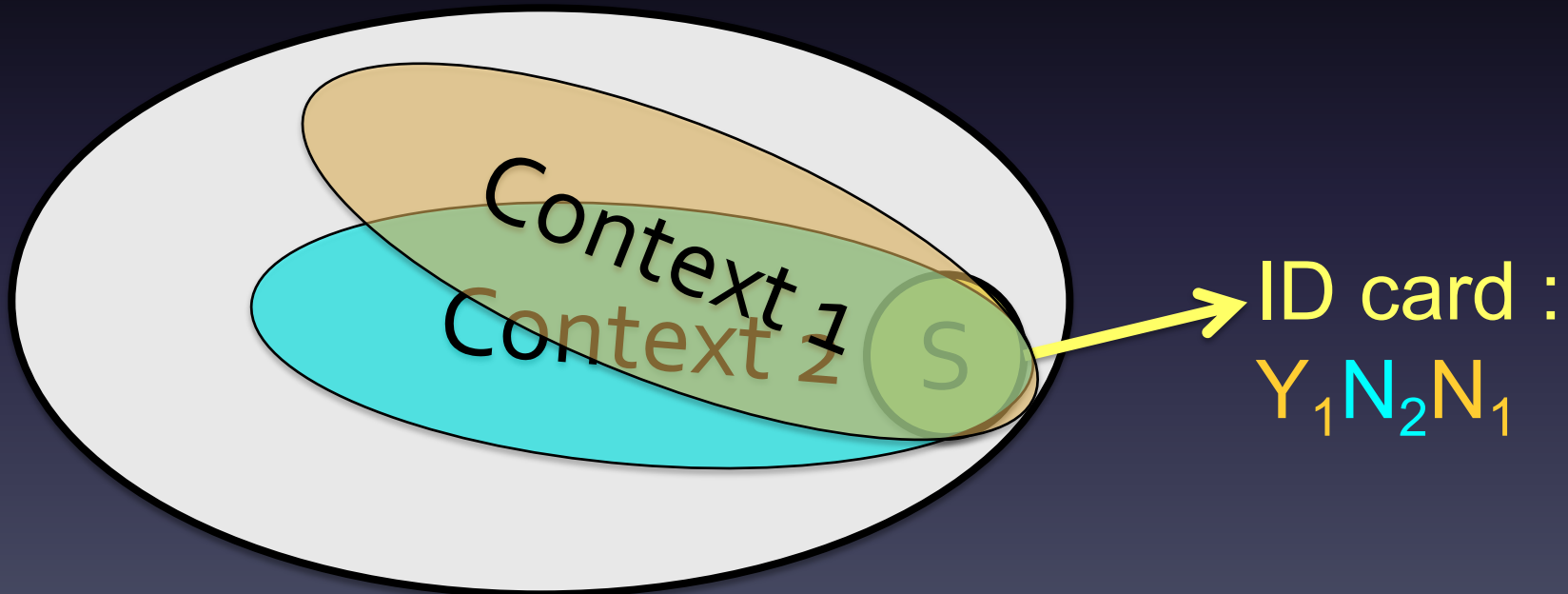
Quantum phenomenology

- Consider a system and a context
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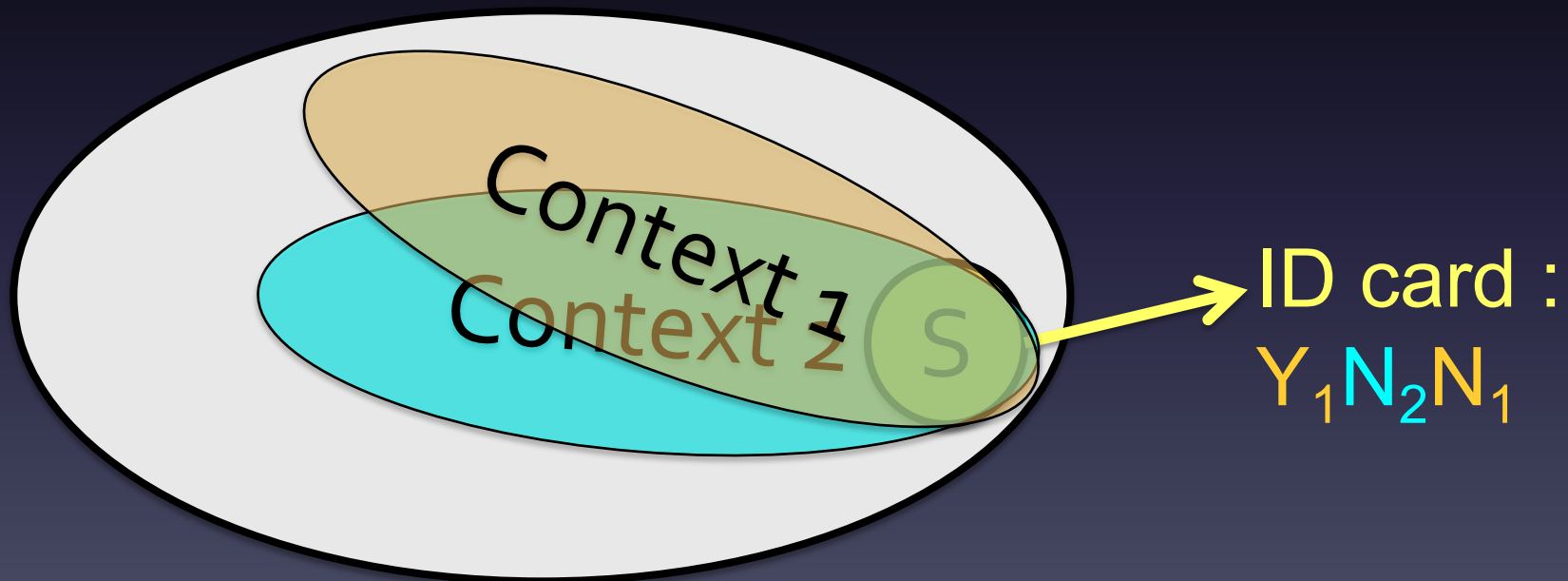
Naïve realist approach

The state pertains to the system alone, the context perturbs the state ☹️



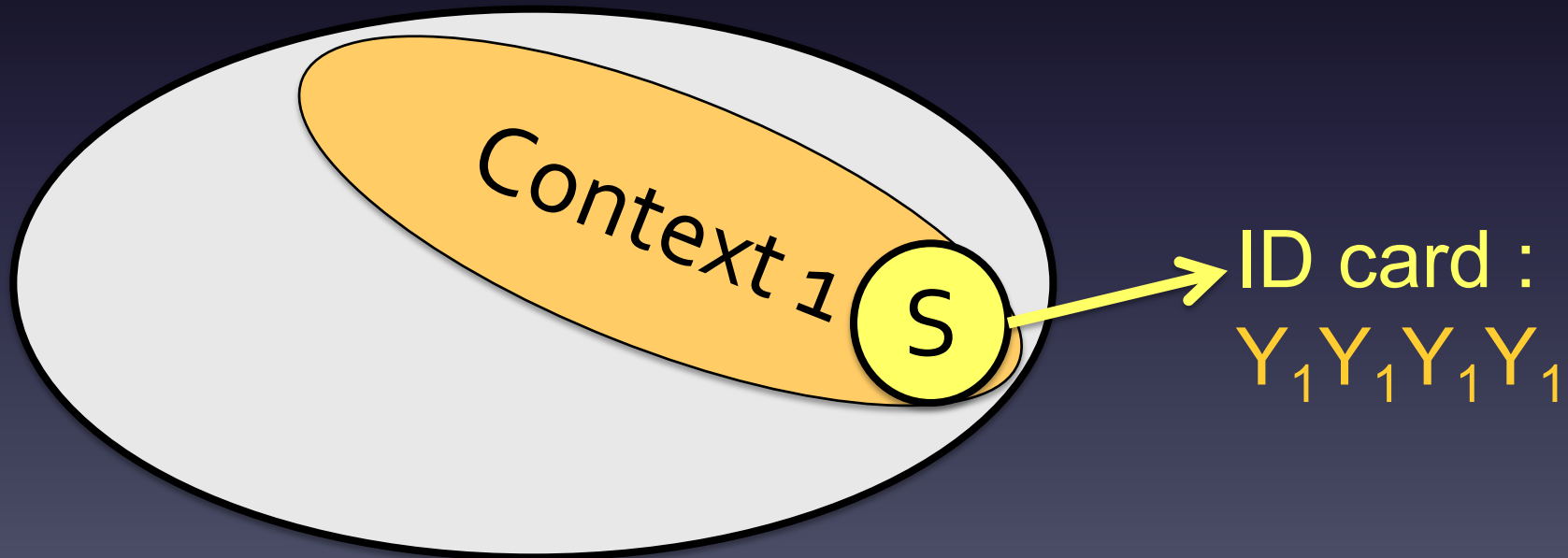
Instrumentalist approach

There are no states, only preparations and measurements 😊



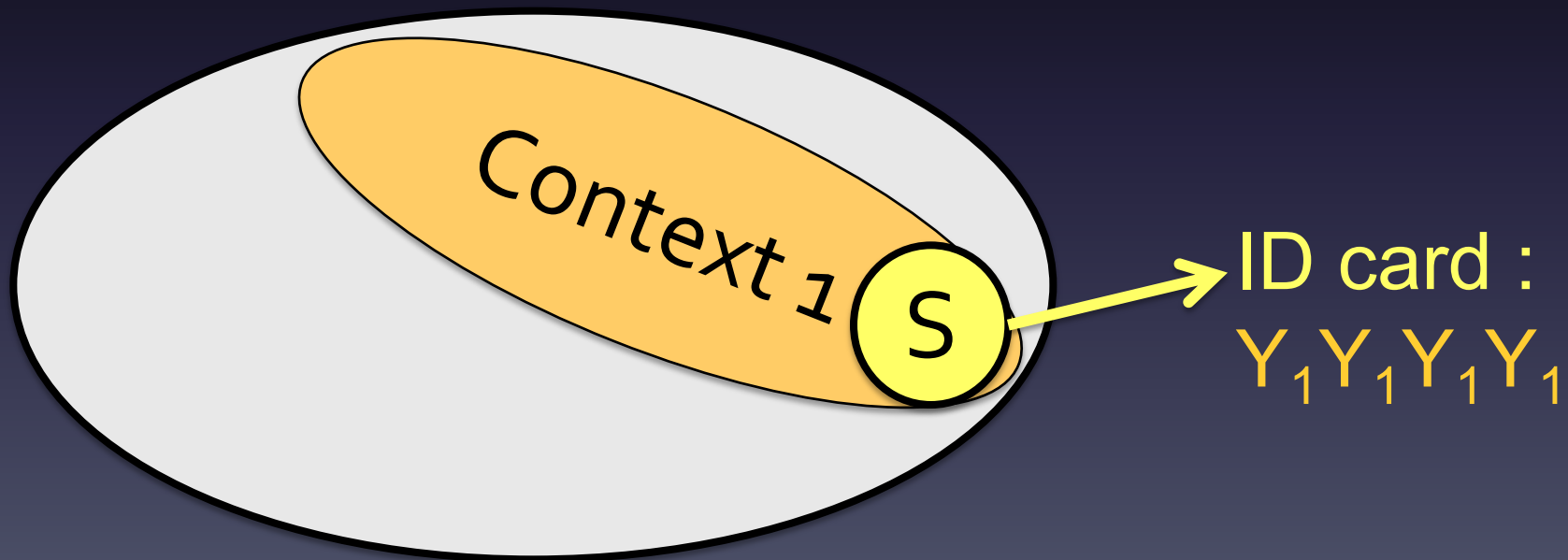
Contextual objectivist approach

« Certainty tracking » : one can obtain repeatably the same answer, **within the same context**
One can upgrade the ID card into a state, **within a given context** 😊



Postulate 1: C-S-M

- A « state » labels both a system and a context
- A contextual state is further called a **modality**

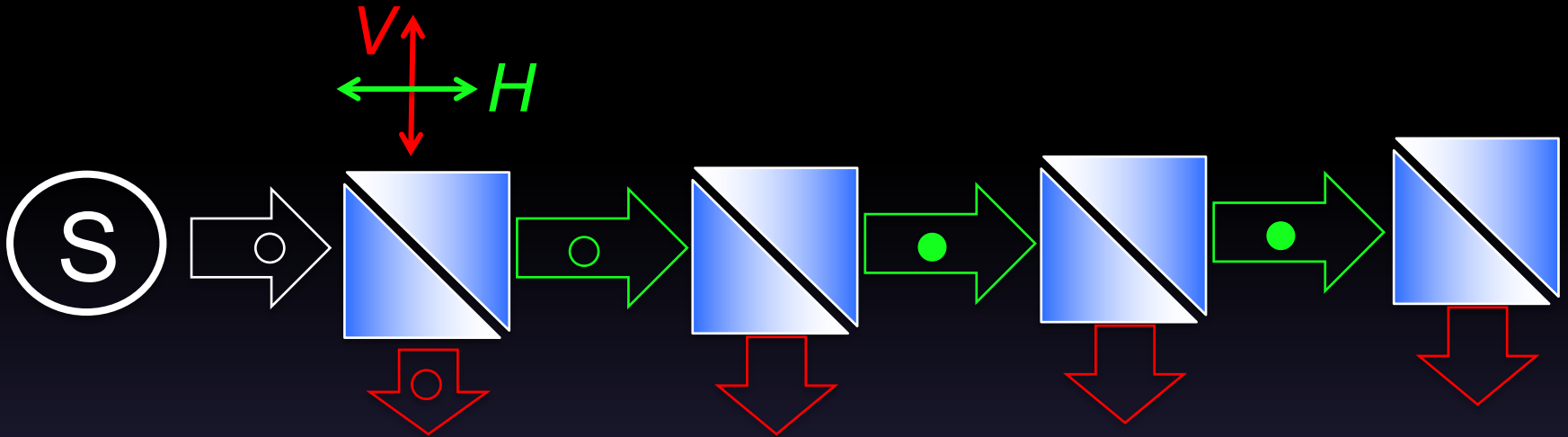


Postulate 1 : C-S-M

A modality has the good properties of a state, but it is contextual

- Take a context prepared in the state C_1
- The sentence « The system and the context are in the modality M » contains a information that can be communicated and on which everyone agrees (intersubjectivity)
- A modality is as objective as a classical state
- « *Contextual objectivity* »

Example



- System = Photon
- Context = PBS
- State of the context = Orientation (H,V) (or (D,A))
- Once a photon is transmitted, it is always transmitted
- « The photon and the context (H,V) are in the modality: Transmitted »

Postulate 1 : C-S-M

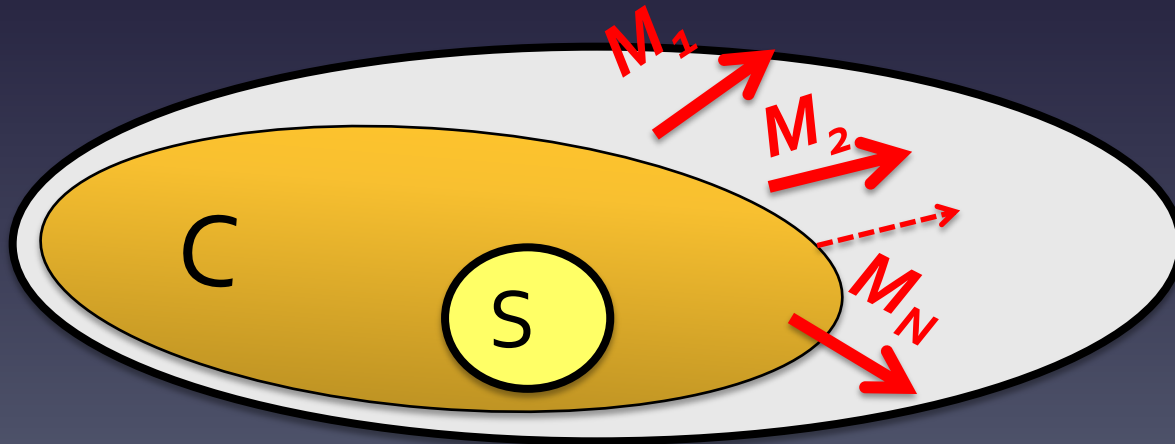
- A « state » labels both a system and a context
- A contextual state is further called a **modality**
- Modalities are as objective as classical states
- *Contextual objectivity*

- Within a given context, modalities are mutually exclusive
- Two modalities pertaining to two different contexts are not necessarily exclusive

Postulate 2 : Elementary systems

Elementary systems are characterized by a fixed, discrete number N of exclusive modalities
(*Quantization/« Discreteness » of modalities*)
 N is independant of the context.

A critical- and textbook - partition of the world : A context around an elementary system with N exclusive modalities



Outline

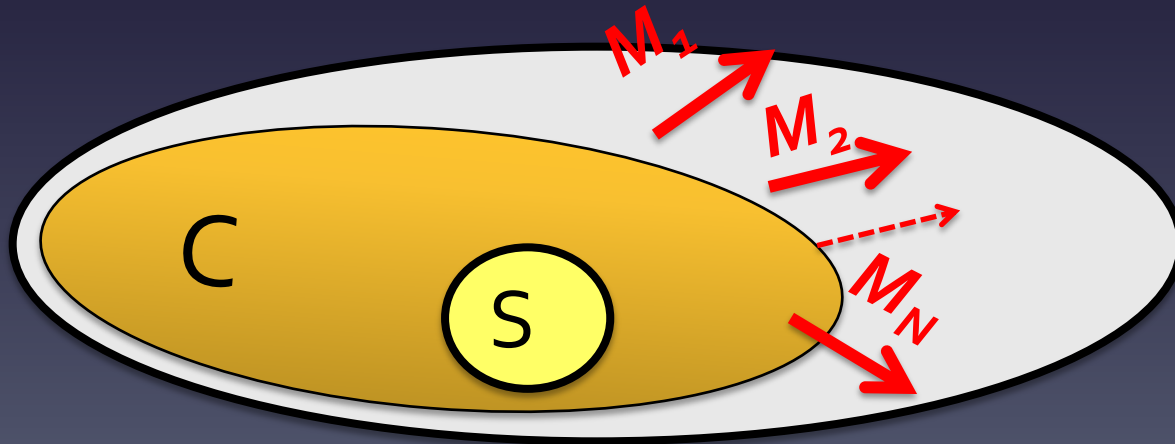
- Contextual approach of reality
- **Randomness in a contextual world**
- Heuristic derivation of the quantum formalism
- Conclusions & outlooks

2 claims

This specific partition of the world leads to

1. Ontological randomness
2. Quantum formalism & quantum randomness

A critical- and textbook - partition of the world : A context around an elementary system with N exclusive modalities



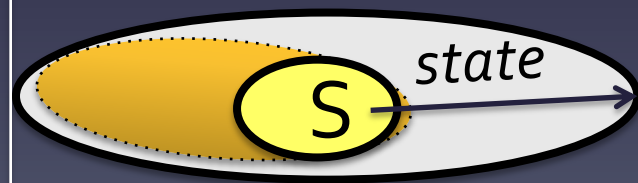
From classical states to modalities

Universality of contextuality

States always show up in a given context

« *Soft* »
contextuality

- Contexts are there, but can be forgotten
- Classical phenomenology



*Quantization of
modalities*

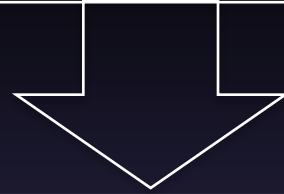
« *Hard* »
contextuality

- Contexts are needed to define a state
- Quantum phenomenology



Claim 1

- Universality of contextuality
- Discreteness of modalities for elementary systems



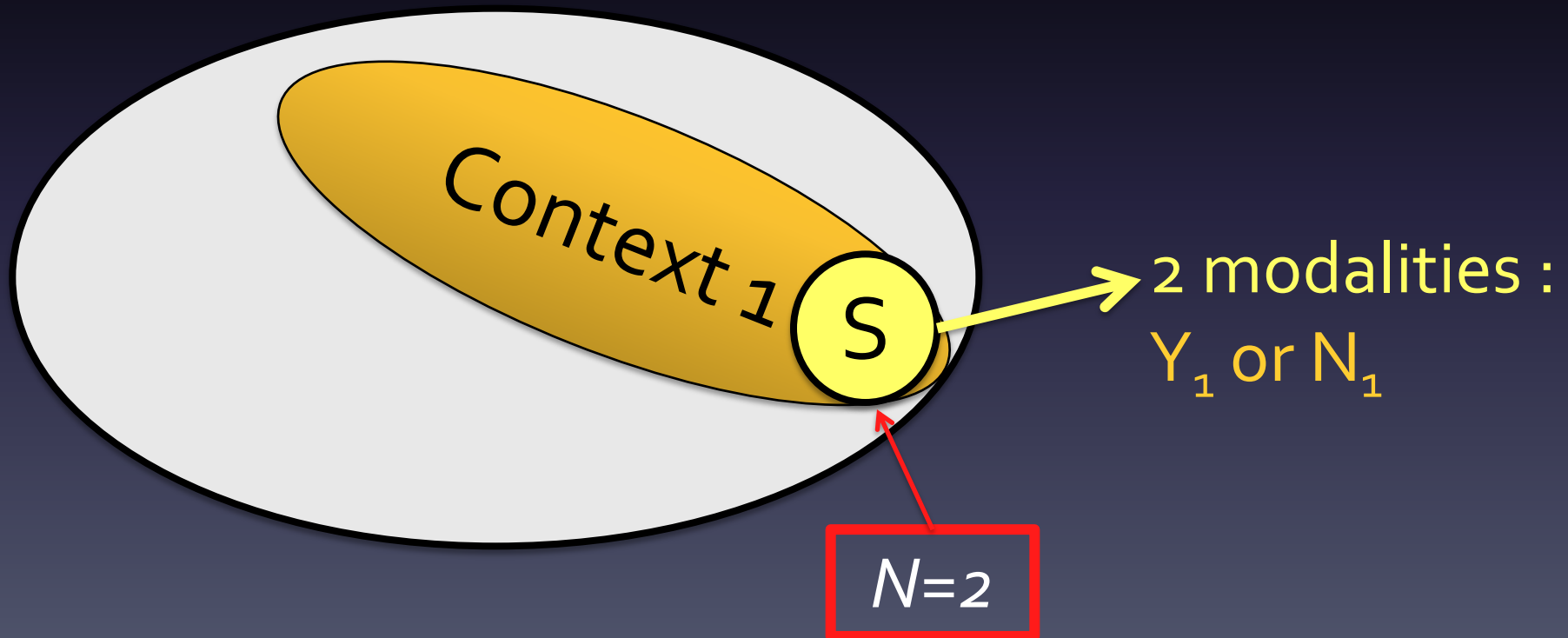
« Hard contextuality »

- Non commutation of the questions
- Unpredictability of the answer as the context changes
 - **Ontological randomness**

Proof

- Elementary system with $N=2$
- Ability to change the context state from C_1 to C_2

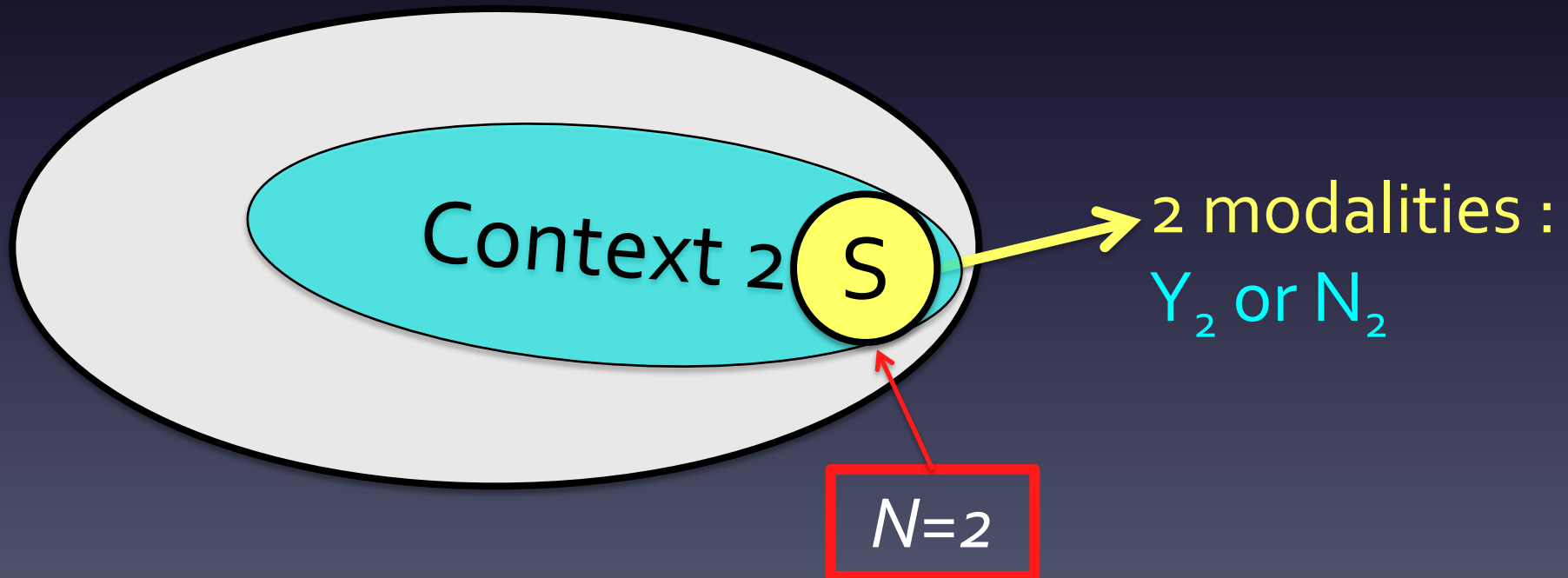
Context 1 : 2 repeatable answers \Rightarrow 2 exclusive modalities



Proof

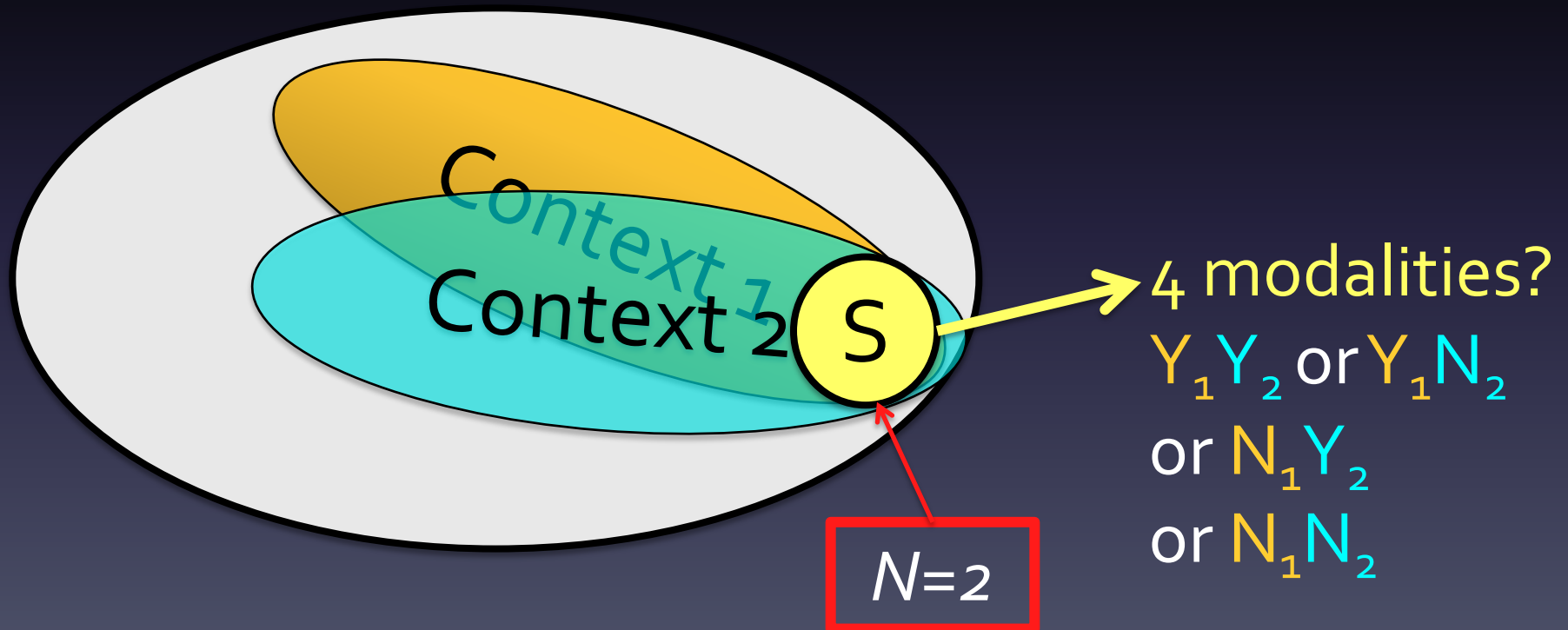
- Elementary system with $N=2$
- Ability to change the context state from C_1 to C_2

Context 2 : 2 repeatable answers \Rightarrow 2 exclusive modalities



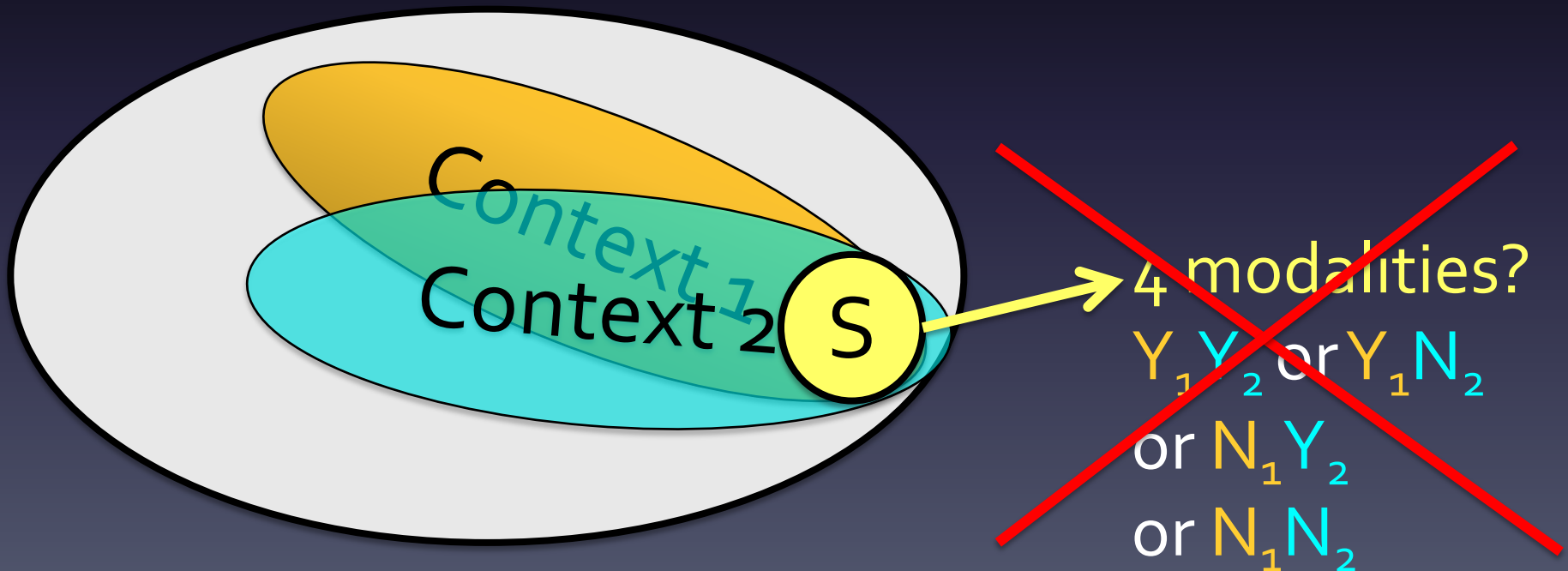
Proof

Back to Context 1 : Can I predict the answers with certainty?



Proof

The answer cannot be predicted, otherwise there
would be 4 exclusive modalities
=> Violation of the quantization postulate



Ontological randomness

- Universal contextuality
- System with discrete modalities
- Ability to change the context state

Conflict: Less repeatable answers allowed, than possible answers to all possible questions

- Non-commutation of the questions
- Unpredictable answers
- **Ontological randomness**

*Core
quantum
features*

Claim 2

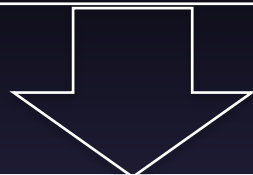
- Universal contextuality
- System with discrete modalities
- Ability to change the context state



- Ontological randomness
- Hard contextuality

Claim 2

- Universal contextuality
- System with discrete modalities
- Ability to change the context state
continuously



To be described: random change of modality

- **Quantum** randomness (Born's rule)
 - Hard contextuality
 - **Quantum formalism**

Outline

- Contextual approach of reality
- Randomness in a contextual world
- **Heuristic derivation of the quantum formalism**
- Conclusions & Outlooks

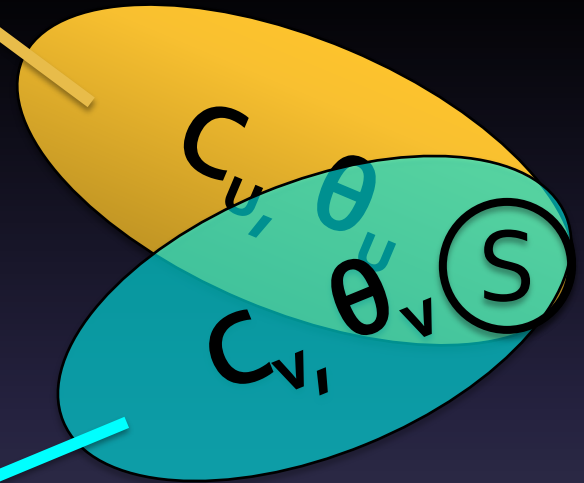
The fundamental quantum event

N modalities $\{u_i\}$



Random change of
modality when the
context is changed

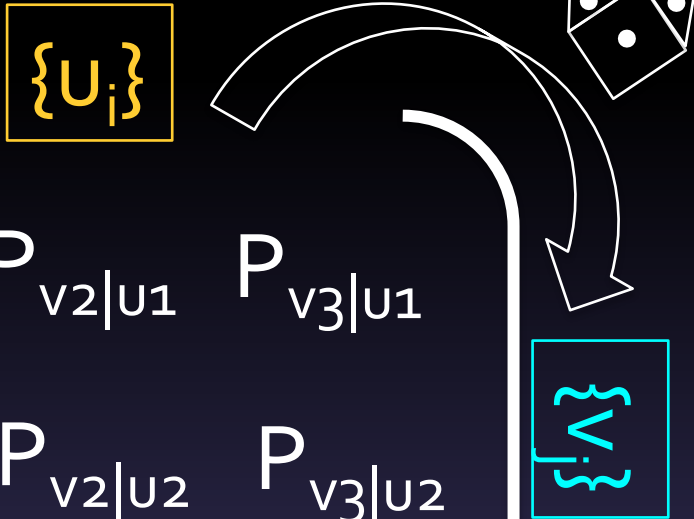
N modalities $\{v_j\}$



Goal of the theory: Describe the event

The fundamental mathematical object

Stochastic probability matrix

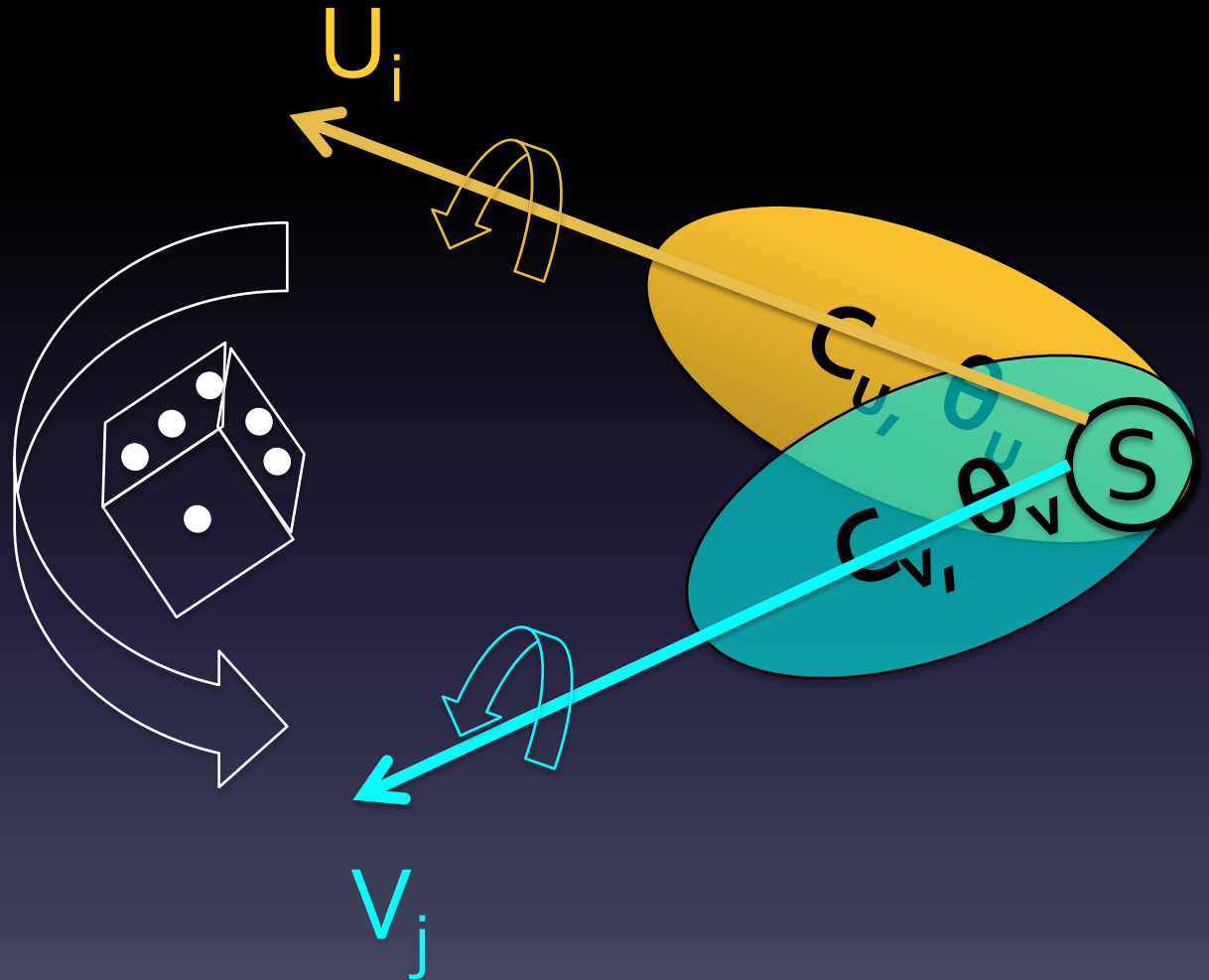
$$\Pi(v|u) = \begin{pmatrix} P_{v_1|u_1} & P_{v_2|u_1} & P_{v_3|u_1} \\ P_{v_1|u_2} & P_{v_2|u_2} & P_{v_3|u_2} \\ P_{v_1|u_3} & P_{v_2|u_3} & P_{v_3|u_3} \end{pmatrix}$$


The diagram illustrates the stochastic probability matrix $\Pi(v|u)$. A yellow box labeled $\{u_i\}$ is positioned above the matrix, and a cyan box labeled $\{v_i\}$ is positioned below it. A curved arrow points from the yellow box to the cyan box, passing over the matrix. A small die icon is also present near the arrow.

Goal of the theory: Model $\Pi(v|u)$

Extra-contextuality of modalities

$P_{v_j|u_i}$
solely
depends
on U_i
and V_j



Step 1: Rewrite Π

$$\Pi(v|u) =$$

$$\begin{pmatrix} P_{v1|u1} & P_{v2|u1} & P_{v3|u1} \\ P_{v1|u2} & P_{v2|u2} & P_{v3|u2} \\ P_{v1|u3} & P_{v2|u3} & P_{v3|u3} \end{pmatrix}$$

- $P_{vj|ui} = \text{Tr}[P_i \Sigma^+ P_j \Sigma]$
- $\Sigma = [P_{vj|ui}^{1/2} \exp(i\phi_{vj|ui})]$
- $\phi_{vj|ui}$ arbitrary phases

$$P_k =$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

k column

k line

Goal: put constraints on these phases

Step 1: Rewrite Π

$$\Pi(v|u) =$$

$$\begin{pmatrix} P_{v1|u1} & P_{v2|u1} & P_{v3|u1} \\ P_{v1|u2} & P_{v2|u2} & P_{v3|u2} \\ P_{v1|u3} & P_{v2|u3} & P_{v3|u3} \end{pmatrix}$$

- $P_{vj|ui} = \text{Tr}[P_i \Sigma + P_j \Sigma]$
- $\Sigma = [P_{vj|ui}^{1/2} \exp(i\phi_{vj|ui})]$

$$P_k =$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

k column

k line

- $\Sigma(C_u, C_v)$
- Contexts pertain to a continuous group
- $\Sigma = 1$ if no change of context
- $\Sigma \rightarrow 1$ if $C_v \rightarrow C_u$

Step 1: Rewrite Π

$$\Pi(v|u) =$$

$$\begin{pmatrix} P_{v1|u1} & P_{v2|u1} & P_{v3|u1} \\ P_{v1|u2} & P_{v2|u2} & P_{v3|u2} \\ P_{v1|u3} & P_{v2|u3} & P_{v3|u3} \end{pmatrix}$$

$$P_k =$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

k column

k line

$$P_{vj|ui} = \text{Tr}[P_i \Sigma^+ P_j \Sigma]$$



Singular values decomposition

- $P_{vj|ui} = \text{Tr}[P'_i R P''_j R]$
- $\{P'_i\}; \{P''_j\}$ projectors
- R real diagonal positive

$$\Sigma = URV^+, \Sigma^+ = VRU^+$$

$$P' = UPU^+, P'' = VPV^+$$

$$U, V \text{ unitaries}$$

Step 2: Call ontology for help

$$P_{v_j|u_i} = \text{Tr}[P'_i R P''_j R]$$

Depends on C_u and C_v

*Identity for
stochastic matrices*




$$P_{v_j|u_i} = \text{Tr}[P'_i R P''_j R]$$

- R, P'_i, P''_j depend on U_i and V_j only
- R, P'_i, P''_j invariant when $C_u \rightarrow C'_u$ and $C_v \rightarrow C'_v$ keeping U_i and V_j unchanged

*Extra-
contextuality
of modalities*

Step 3: Chase the contradiction

Wanted: $R(\mathbf{U}_i, \mathbf{V}_j)$

- $\text{Tr}[R^2 P'_k] = 1$
For each k  $\text{Tr}[(R^2 - 1)P'_k] = 0$
N linear equations
 $D = \text{Det}[|\mathbf{U}_{m,n}|^2]$

Either $R=1$, or $D=0$

- Suppose $R(\mathbf{C}_u, \mathbf{C}_v) \neq 1 \Rightarrow D(\mathbf{C}_u, \mathbf{C}_v) = 0$
- $\mathbf{C}_u \rightarrow \mathbf{C}'_u; \mathbf{C}_v \rightarrow \mathbf{C}'_v \Rightarrow D \neq 0 \Rightarrow R(\mathbf{C}'_u, \mathbf{C}'_v) = 1$
- R depends on the whole contexts

ABSURD $\Rightarrow R=1$

Step 4: Unitary matrices

$$P_{v_j|u_i} = \text{Tr}[P_i \Sigma^+ P_j \Sigma]$$
$$\Sigma = [P_{v_j|u_i}^{1/2} \exp(i\phi_{v_j|u_i})]$$

$$\Sigma = URV^+, \Sigma^+ = VRU^+$$

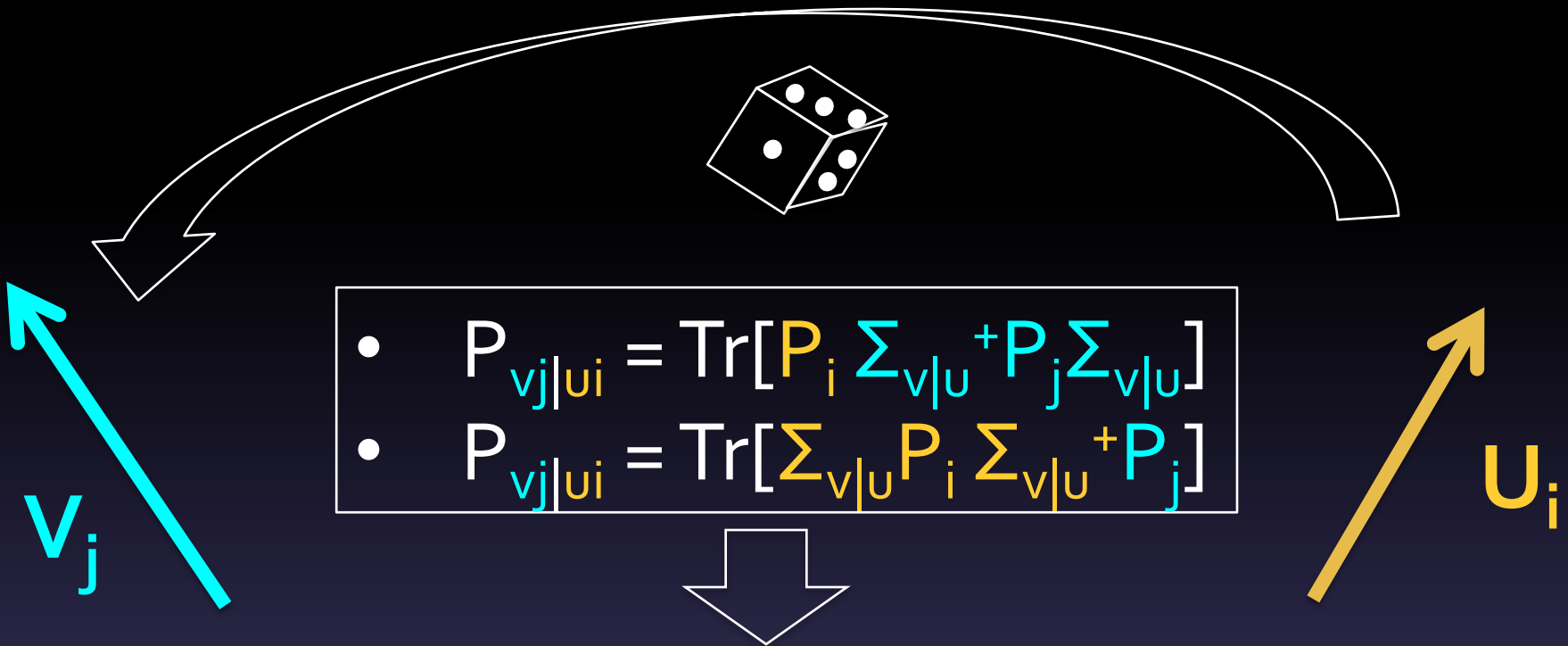
U, V unitaries

$$R=1 \Rightarrow \Sigma = \Sigma^+ = \Sigma^{-1}$$

Real matrices?

Continuity of contexts
Continuous path relating identity
and permutation
 $\Sigma = \text{complex, unitary matrix}$

And finally: Usual quantum formalism



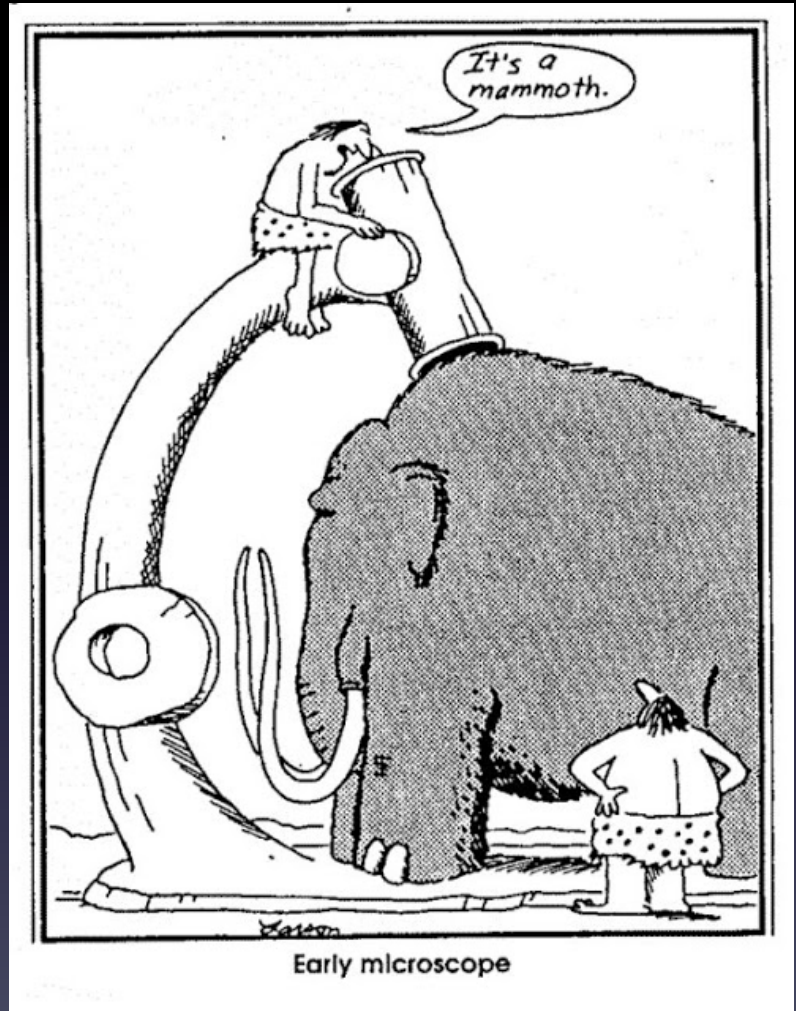
- U_i, V_j = Rays in a Hilbert space
- Change of context = $\Sigma_{v|u}$ unitary
- Probabilities follow Born's rule

Outline

- Contextual approach of reality
- Randomness in a contextual world
- Heuristic derivation of the quantum formalism
- **Conclusions & outlooks**

Origin of randomness in a contextual world

- Context around a system
 - Reduce the number of modalities of the system
- Elementary system

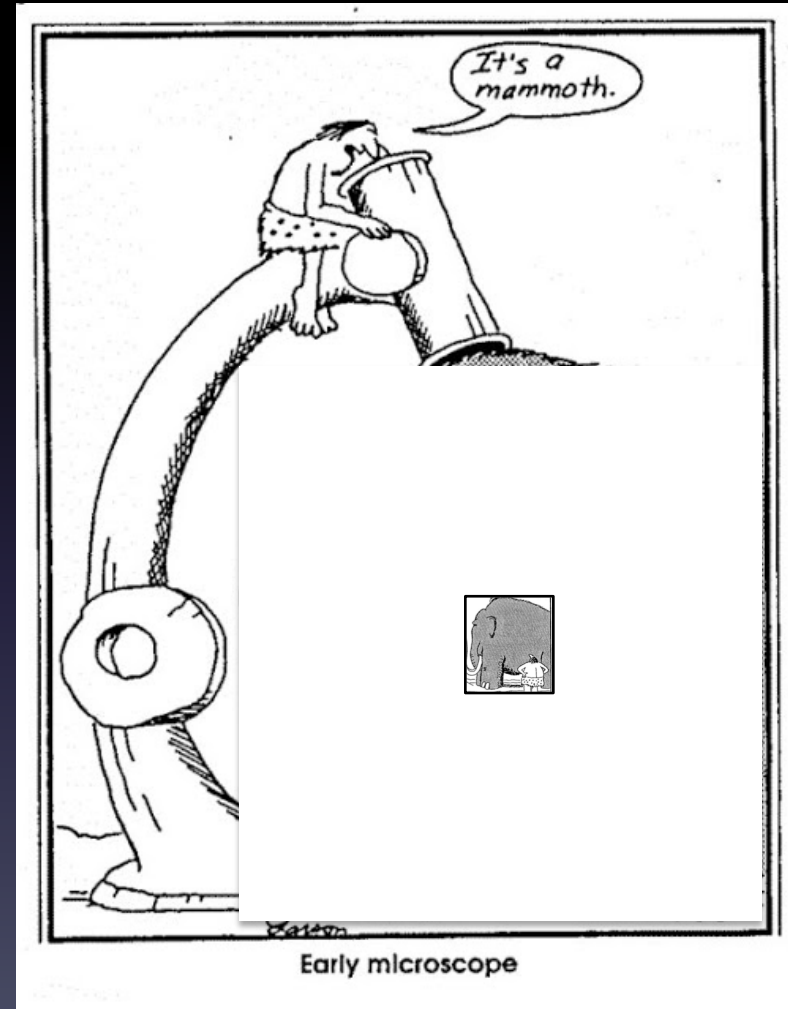


Origin of randomness in a contextual world

- Context around an elementary system with N modalities

Quantum randomness appears, because

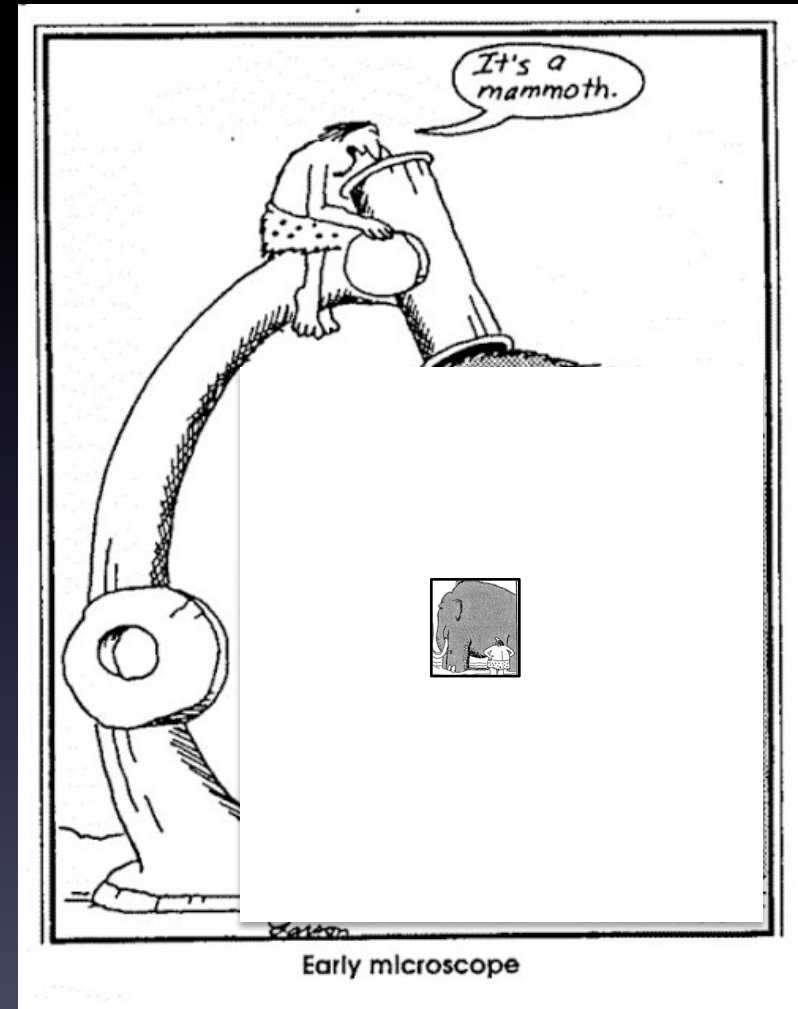
- There are less repeatable that possible answers (to all possible questions)
- **The context/system interface is a condition of the quantum formalism**



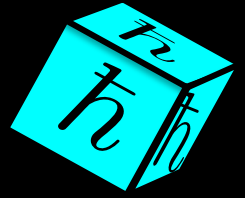
Origin of randomness in a contextual world

Top-down approach

- The context is always already there
- The system will never « swallow the context »
- No emergence of the classical (from the quantum)
- Challenges classical reductionism



The 2 alea



- Epistemological randomness
- Information loss due to coarse-graining
- Bottom up approach from microscopic to macroscopic
- Not absolute
- « Cured » by reductionism = Program of statistical physics


- Ontological randomness
- Unpredictability due to contextuality + quantization
- Top down approach from macroscopic to microscopic
- Absolute and irreducible
- Soil of the quantum formalism

About reversed hierarchies

1. Ontology *first*, formalism second
2. Phenomena *first*, « ψ » second
3. Non-unitarity *first*, unitarity second
(or Randomness first, determinism second)
4. Discreteness *first*, interferences second

Ontology first, formalism second

Quantum phenomenology
=> Ontological postulates
Contextuality/Discreteness/Continuity

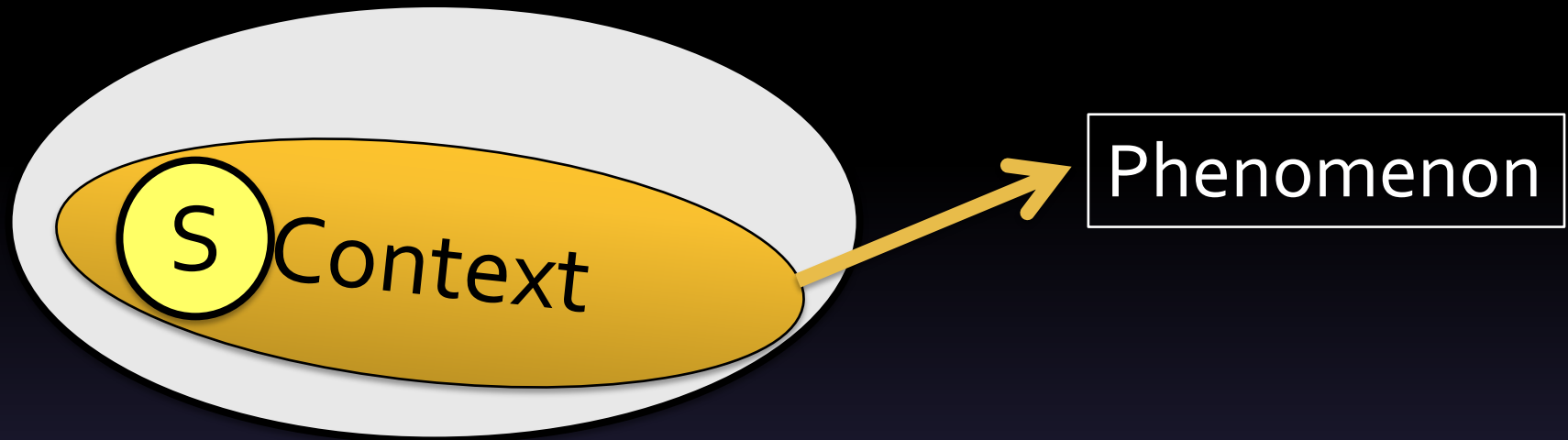


Core quantum features
Ontological and quantum randomness



Formalism
Born's rule
Hilbert spaces

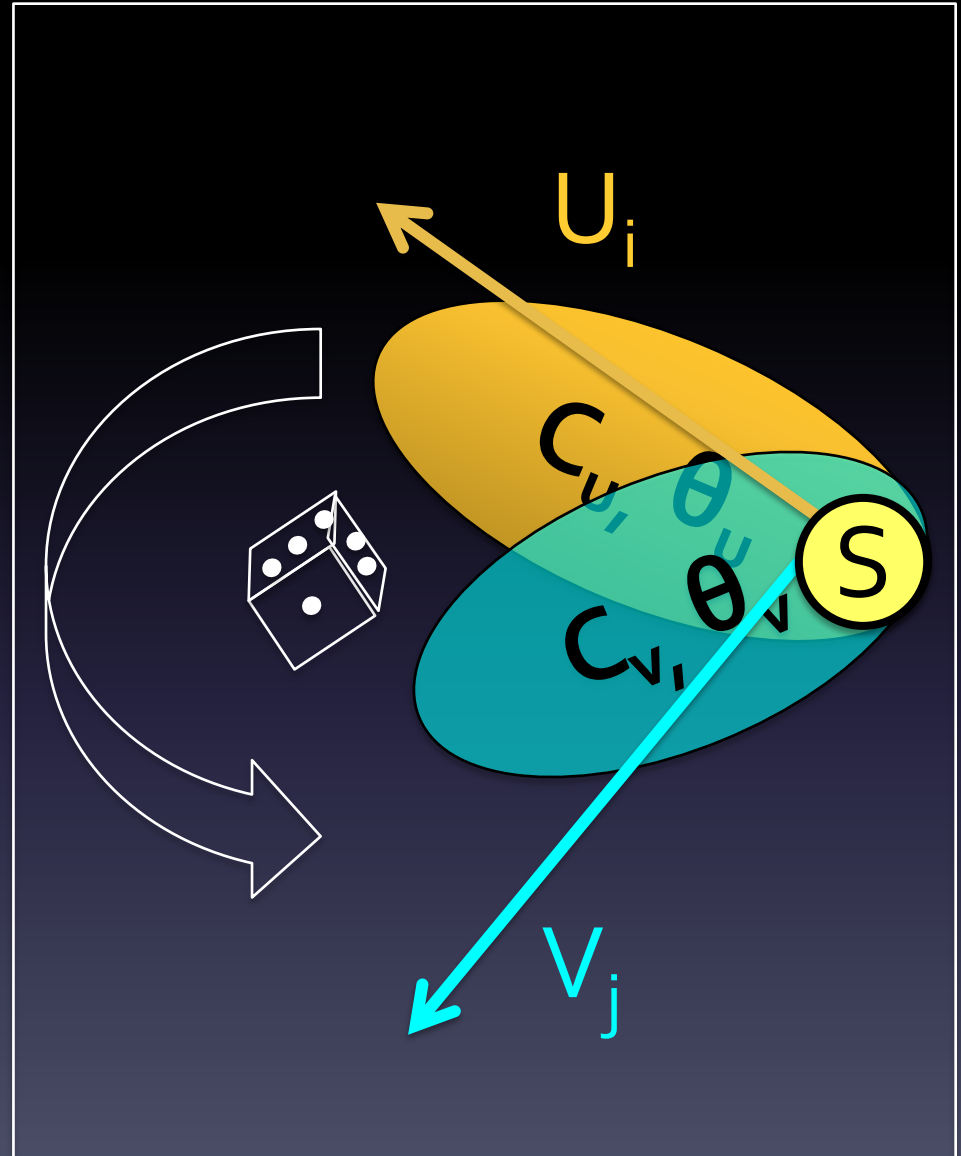
Phenomena first, « ψ » second



- The CSM approach of reality is based on repeatable, certain, actual, objective events
- What is **real** is the modality-phenomenon, not the hidden wave function
- *Price to pay: Modalities are contextual*

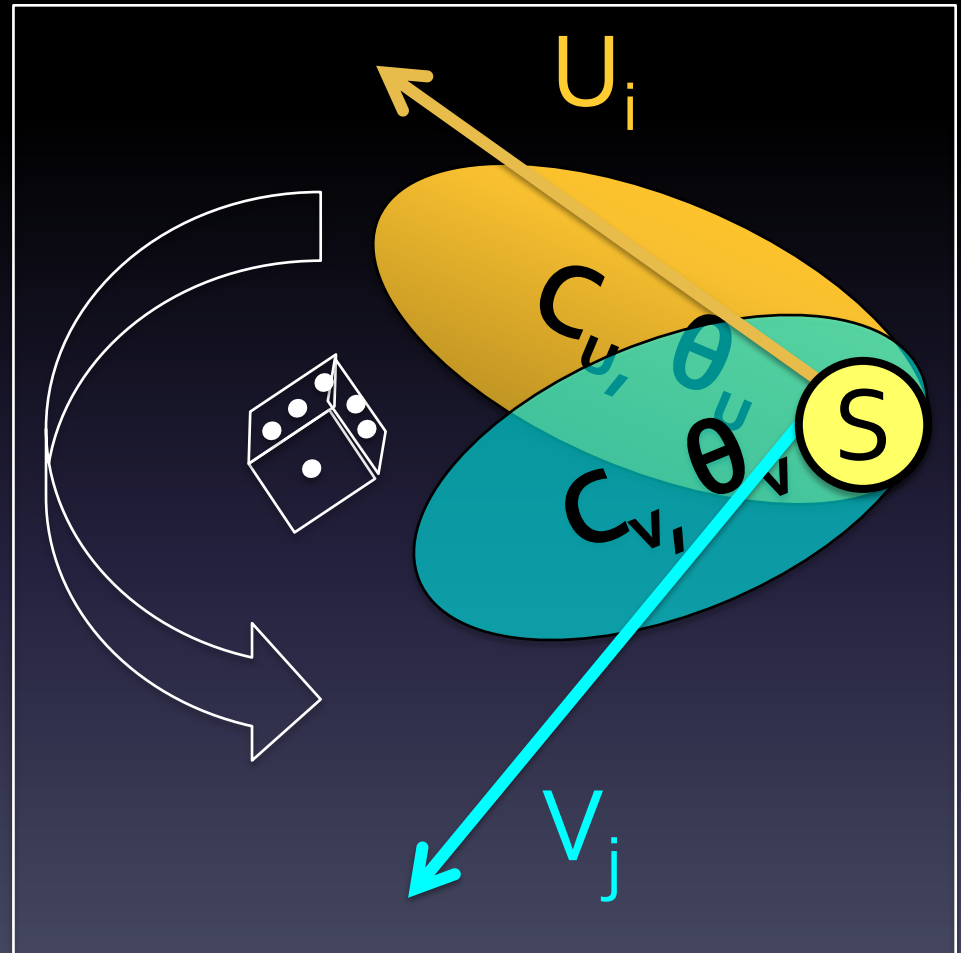
Non-unitarity first, unitarity second

- Fundamental quantum event to be described by the theory= the random change of probability (Non-unitary)
- Unitary transformations describe the change of context

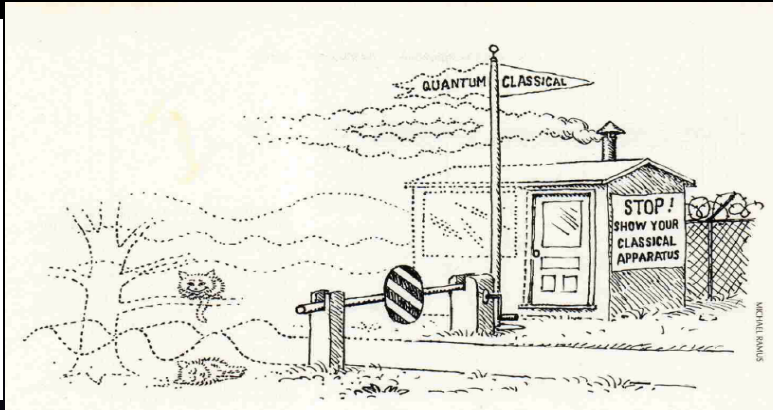
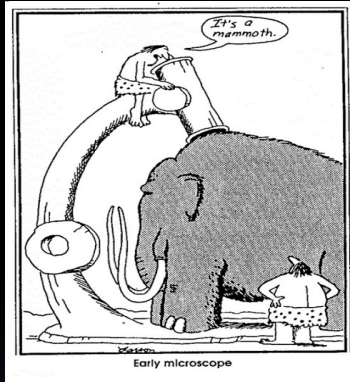


Quantization first, interferences second

- The random change of modality is due to quantization
- To describe context changes, unitary matrices are needed \Rightarrow complex numbers \Rightarrow interferences

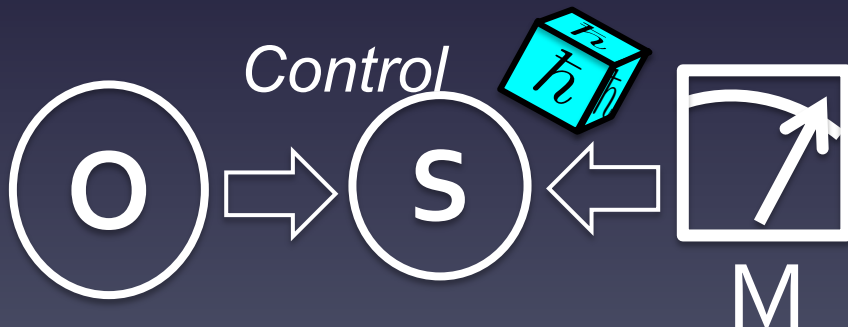
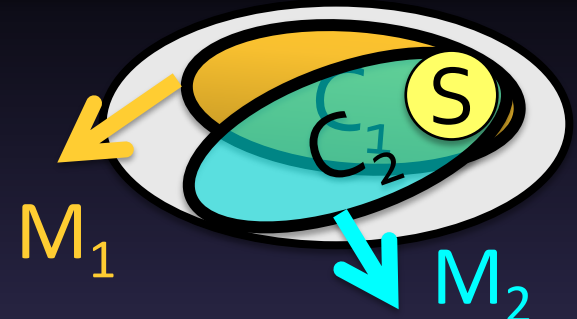


Some outlooks



- Get CSM out of the lab!
- Mathematical origin of the cut?

- Rethink epistemology vs ontology, given contextual objectivity



- Rebuilding quantum thermodynamics on quantum randomness
- What is contextual thermodynamics?