# PREDICTABILITY AND PHASE SPACE RELATIONSHIPS OF CLIMATIC CHANGES, ENSO AND TUNA BIOMASS ON THE EASTERN PACIFIC OCEAN.

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**Abstract**
Dynamical system in terms of phase space are defined by the collection of variables needed to determine the state of the system at a given time, where sensitive dependence on initial conditions is said to be chaotic. The earth's atmosphere and its climate can be considered a chaotic system and therefore it should be treated as a dynamic system.
Regime behavior can also significantly impact predictability, it occurs when a system spends large amounts of time in a localized region of the attractor, followed by relative rapid transitions to other regions of the attractor.The goal to predict the sequences of the regimes and their durations has not been successful.
Sensitive dependence on initial conditions of chaotic systems with their largest Lyapunov exponent positive impose a practical limit on predictability. It should be noted also that the relative weak non linearity of tropical systems allow successful predictions of ENSO and Tuna Biomass in the Easthern Pacific and this prediction provides some degree of global climate predictions.

**Key words** –Tuna Biomass, attractor, Lyapunov, predictions

## INTRODUCTION

There is a general aggreement in the meteorological community that the behavior of the atmosphere is chaotic; its sensibility to initial conditions is such that its predictibility beyond a certain time range is almost nil. Chaotic behaviors are observed in those systems containing a few dynamically relevant variables. Consequently, it has recently been suggested that a few variables may be sufficient in order to model the atmosphere´s dynamics. This is one of the most relevant results in dynamic systems theory. That is the complex behaviour and impredictability of the atmosphere is not necessarily due to the presence of a great number of the degrees of freedom,

Thus Nicolis and Nicolis (1986) have noted that the world´s climate one million years ago was limited to one attractor of three dimensions. If this statement is correct we ought to be able to simulate the atmosphere with equations having four independent variables.

The problem would consist in selecting the adequate prediction that should be representative of the processes affecting such variables. Not withstanding the difficulty in identifying the appropriate variables, the question is to determine whether such complex system as the atmosphere is likely to be represented by an attractor of low dimension in

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which this dimension number could function as a guide in the development of new theories of climate.

The analysis also suggest that climatic variability is the manifestation of chaos dynamics described by an attractor of fractal dimensionality. Possibility of applying certain ideas of dynamic systems theory to the study of global climate suggest that a strange attractor may be characterized by a finite time series obtained from samples of the dynamic system. Moreover, the possible existence of a “strange attractor” in the system offers the option and possibility of having this information directly from the data, instead of obtaining them indirectly from the modeling process. This within the adequate sampling time to draw relyable conclusions, Hastings et al (1993).

The idea of calculating the dimension of a strange attractor would thus have a limited use in those systems displaying several degrees of freedom. However an attractor of five to ten dimensions in series of ten to a thousand years of information would be consistent with the known climate dynamics.

If this characteristics were found in the system one would expect that such system may be described with a very simple model having few degrees of liberty. Recent developments in dynamic systems theory show that if a system develops in a deterministic mode, its typical trajectory remains in a low dimension of the total available space phases. Moreover, strange attractors are now found in which the trajectory remains as a fractal characterized by an incomplete dimension.

New approaches for analyzing time series data are based on positive Lyapunov exponents to denote the presence of chaos, Sugihara and May (1990). Positive Lyapunov exponents imply noise amplification and the spontaneous production of new macroscopic information through the amplification of small fluctuations. Where such behavior can be easily confused with deterministic chaos, Hastings et al (1993)

A nonchaotic system is a noise muffler: the effects of external perturbations decay asymptotically to zero over time. Here the amount of time until the effects of a given perturbation die out is roughly proportional to the inverse of the Lyapunov value.

Deterministic chaos refers to nonlineal difference or differential equations without any randomness. Deliberate oversimplifications tend to omit any stochastic factors affecting the system. Both the system itself and external perturbations contribute to the system’s unpredictability, , Sugihara and May (1990).

Any unpredictability is solely due to random perturbations.

The use of nonlinear prediction schemes, especially dynamic systems models, will necessarily be the required step to take in the future in order to understand and quantify the complexity of weather and climate systems including climatic change.

**METODOLOGIA**

Predictability of dynamic systems is closely related to the problem of stability. It is derived from phase space trajectories of weather and climate variables which evolve on attractors, and are accompanied by a growth of errors, which ultimately leads to limits of the predictability.

Local stability properties as the overall stability can have a dramatic physical effect and can cause otherwise imperceptible ambient noise to be amplified to macroscopic proportions. Local instabilities in phase space cause fluctuations to be temporarily amplified. When a

noise is applied, a region of local instability near a fixed point amplifies any external noise, carrying the orbit past that fixed point, May (1976), May and Oster (1976).
We propose the next model in order to analize this:

**THE MODEL**
Suppose the climate is given by the multivariables dynamics of m equations.
$dN_i/dt = fi\ (N_1(t), N_2(t), \ldots, N_m(t))$ 1
Where the rate of change of the ith variable at time t is given by some nonlinear function fi, of all the relevant interacting variables.
The equilibrium, N*, follow from the m algebraic equations obtained by putting all rates of change equal to zero.
$F_i\ (N^{*1}, N^{*2}, \ldots, N^{*m}) = 0$ 2
Expanding about this equilibrium, for each variable we write.
$Ni(t) = N^*i + ni\ (t)$
Where xi measures the initial small perturbation to the ith variable.
A linearized approximation is obtained, as:
$ni(t) = \sum Cij \exp(\lambda jt)$
$d\ ni\ (t)/dt = \sum aij\ xj\ (t)$ 4
This set of m equations describes the climate dynamics in the neigborhood of the equilibrium point. Equivalently, we may write in matrix notation.
$d\ \mathbf{X}\ \mathbf{(t)}/dt = \mathbf{A}\ \mathbf{X}\ \mathbf{(t)}$ **5**
Here **X** is the mx1 column matrix of the ni and **A** is the mxm “ Jacobian matrix “ whose elements aij describe the effect of variable j upon variable i near equilibrium.
The elements aij depend both on the details of the original equations (1) and on the values of this equilibrium climatic parameters, according to:
$aij = (\partial fi / \partial Nj)^*$ 6
The partial derivatives denoting the derivatives of fi keeping all variables except Nj, constant, are to be evaluated with all of them having their equilibrium values.
The m constants λj which caracterize the temporal behavior of the system are the eigenvalues of the matrix A.
$(A - \lambda I)\ X\ (t) = 0$ 7
Here I is the mxm unit matrix. This set of equations posseses a non-trivial solution if and only if the determinant vanishes.
$Det\ I\ A - \lambda\ I\ I = 0$ 8
This is in effect a mth order polynomial equation in λ, and it determines the eigenvalues λ of the matrix A.
They may in general be complex numbers, $\lambda = \zeta + I\ \xi$ , the real part ζ produces exponential growth or decay, and the imaginary part ξ produces sinusoidal oscillation.
It is clear that the perturbations to the equilibrium values will die away in time if and only if, all eigenvalues λ have negative real parts. If any one eigenvalue has a positive real part, the exponential factor will grow ever larger as time goes on, and consequently the equilibrium is unstable, Ritter et al (1988),Ritter et al( 2004), figure (4).
The special case of neutral stability is attained if one or more eigenvalues are purely imaginary numbers and the rest have negative real parts.

An equilibrium configuration in the multiclimatic variable system will have neighborhood stability, if and only if, all eigenvalues of the “Jacobian matrix” A, lie in the left-hand of the plane of complex numbers.
It is convenient to define $\Lambda$ as minus the largest real part of all eigenvalues of the “Jacobian matrix”.

$- \Lambda = \text{Real}(\lambda)\max$ 9

The stability criterion then becomes

$\Lambda > 0$ 10

Eventually terms of second order and higher become important, and nonlinearities decide whether the perturbations will grow until extinctions are produced, or whether the system may settle into some limit cycle.
Likewise even if the equilibrium point is stable to small perturbations, its response to severe buffetings is not necessarily known.
The Lotka-Volterra equations are one particular case of the general formal equations (1). Equations of this type (logistic) faithfully characterize the stability propierties of a much wider class of models.
This means we have linear combinations of purely oscillatory factors, which is to say linear combinations of cos (wt) and sen (wt), with perturbations leading to undamped pure oscillations of frequency w or period $2\pi$ /w.
The direct analogue of Lotka Volterra equation is

$dHi(t)/dt = Hi(t)(ai - \sum \alpha ij\, Pj\,(t))$

$dPi(t)/dt = Pi(t)(- bi + \sum \beta ijHj(t))$ 11

With i = 1,2,…., n. All the parameters ai, bi, $\alpha$ij, $\beta$ij are positive numbers.
The “Jacobian matrix” which characterizes the stability of this multivariable system is now obtained by applying aij to the equations 11.
It is evidently a 2nx2n matrix, partitioned into four nxn blocks.
Thus one necessary, but not sufficient, condition for neighborhood stability which must be satisfied if all the eigenvalues are to have negative real parts is:

Trace $A < 0$ 12

In the especial case of Trace A = 0, either at least one eigenvalue must lie in the right half plane (unstable system), or all eigenvalues must be purely imaginary (neutral stability).
If A is a mxm matrix, the equation; det I A - $\lambda$I I = 0, for the eigenvalues $\lambda$ comes down to an mth order polynomial equation. A formal general expression call the Routh-Hurwitz criterion, can now be written down, giving constraints on the coefficients, which are necessary and sufficient to ensure all eigenvalues lie in the left hanlf complex plane.
Particularly interesting is the general class of mxm matrices whose rows are cyclic permutations of the firs one. The eigenvalues $\lambda$k, are given by the expressions:

$\lambda k = \sum Cl \exp(2\pi i/m(kl))$ 13

We can approximate the dynamics of the Poincare map by considering only one variable, where the slope of the function relating x(t+1) to x(t) is large in magnitude and the discrete time logistic function lead to chaotic behavior if the functions are steep enough.The local stabilities of the climate evolution are determined by the eigevalues or characteristic exponents, aij, of the Jacobian matrix where in general, the rate of exponential growth of an infinitesimal vector $\delta x(t)$ in the n-dimensional phase space is given by the largest of the Lyapunov characteristic exponent. Thus the growth rate of the phase space is the growth



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rate of the Jacobian determinant and given by the sum of all n eigenvalues. The dynamical system (1) for n variables can be transformed to a single nonlinear differential equation of nth order by eliminating all but one of the variables. This is plausible because for time lags larger than the autocorrelation of the time series tends to zero, the data become linearly independent.

The characteristic exponent or eigenvalue L is defined by the Jacobian:$L= df/dx/x0$ or $d\ln\delta x/dt$.

Whereas neighborhood is concerned with an isolated singularity, global analysis is concerned with the behavior of the system over the entire space.

We can classify singularities in terms of the dynamical behavior of the system near the singularity. Based on the configuration of the two eigenvalues, we can tell which type of system applies to a particular system of differential equations.

If one eigenvalue is positive ( real part) and the other negative, we have a saddle point.

Assumed zero imaginary parts for the eigenvalues in the case in which both eigenvalues have nonzero imaginary parts. With nonzero imaginary parts we are dealing with an oscillatory system, If both real parts are positive, we have a stable focus. If both real parts are positive, we have unstable focus. If both real parts are zero, we have centers of neutrally stable oscillations, Vandermeer (1972).

**RESULTS AND CONCLUSIONS**

The tuna fish population of the Eastern Pacific Ocean (EPO), shows a high sensitivity to ocean temperatures. This fact has been used as a biological index to detect the El Niño phenomenon, Arntz et al (1996), (Suarez et al 2002).

Tuna biomass reacts to El Niño by a drastic population reduction and show recovery. Strong Enso events have a clear influence on tuna populations. In other words El Niño causes an initial reduction in tuna biomass; however once the El Niño is gone the system goes through a rejuvenation process increasing considerably the population biomass ( Suarez et al (2004), Ritter et al.(in press).

The historic tuna biomass in the EPO and the ENSO phenomenon may be easily simulated and forecasted by applying statistical methods to time series, to models such as the ARIMA e.g. Suarez et al (2004),(fig. 2), or dynamic systems methodologies such as neuronal networks in space phases that clearly show a closed toroidal diagram (fig., 3). This means that variables are acting in the presence of periodicities differing only in their frequencies. These methodologies have made possible to issue predictions six months in advance of the event during this year, Suarez et al (2004). These predictions have been possible by the presence of warm waters observed during the last decade which suggests a linearity effect in the system, Chaves et al. (2003), Trenbert (1990),Cobb et al. (2003), Graham(1995).

Predicting how ENSO will change under continued greenhouse forcing, yield a broad range of possible results, where the mean state of the tropical Pacific climate system is know to vary over decadal timescales.

It is possible that with the beginning of a cool phase decade, detected by an increasing amount of anchovets and a drop in the sardine population, this characteristic could be lost and result in nonlinear properties with less forecast possibilities and a greater presence of extreme events forecast.

The recent upward trends in global temperature have been caused by an active tropical hydrologic cycle driven by increasing tropical ocean temperatures, suggesting that an

enhanced tropical hydrologic cycle are similar in importance to the early manifestations of the climatic response to increasing concentrations of greenhouse gases,Graham (1995).
The diagram in figure 4,Ritter et al. (in press) derived from the manifestation of the logistic model allows to determine the eigenvalues of vectors in the space phases. The relevance of this diagram is the correspondence of the strongest El Niño events occurs when the tuna biomass vectors invade the chaotic zone which is linked to the maximum eigenvalue of the system, table (1).
Global temperatures anomalies in the graph (fig. 5) show multiple attractors, as expected from complex system´s manifestations, it is possible that these attractors are shown in increasing mode due to increasing emissions of CO2 at the global scale. The temperature curls are also observed in the tuna biomass in the space phases in case of an El Niño condition. However transition between attractors, remain a mystery since it has not been possible up to now to find a clear explanation of its manifestation.
Application of ARIMA statistical process to temperature data (fig. 6) for prediction purposes with the tuna biomass periodicities, shows a clear trend for cooler temperatures in the future, in aggreement with those researchers that expected a decade of cooler oceanic temperatures,Chaves et al (2003), Graham (1995), Trenberth (1990), Cobb et al (2003),
This prognosis is expected to be close to reality if the system tends to conserve its linearity. Finally, of one thing we are certain and that is that we are heading toward a world hiding many surprising situations more difficult to predict. Sugihara and May(1990).

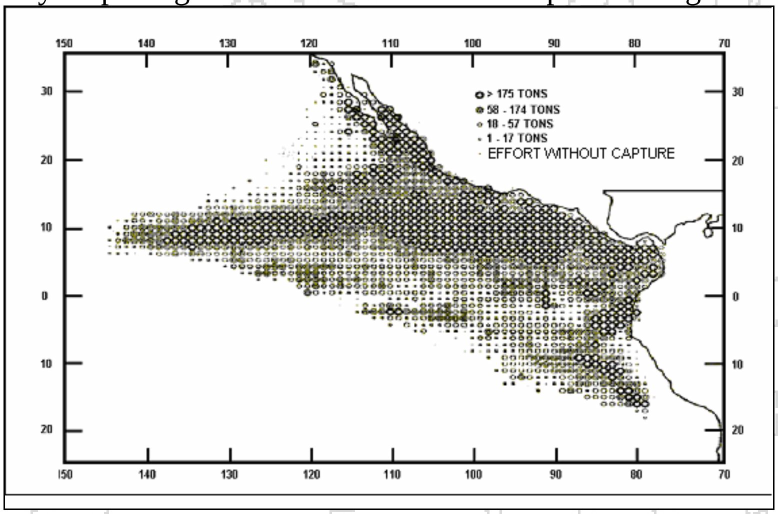


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Figure 1. The Eastern Pacific Ocean and the mean captures during 1979-1993 (taken fromIATTC, 1994)



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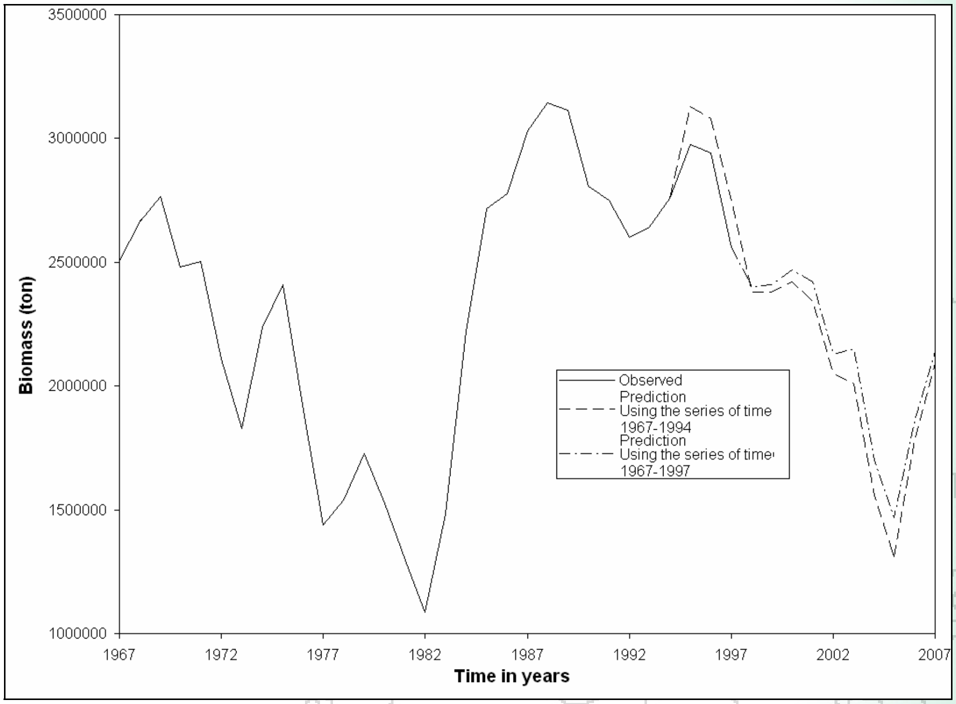


Figure 2 . Average annual biomass of yellowfin tuna fish from the Eastern Pacific Ocean, as (o) observed and (-) estimated by the ARIMA model (0,0,0,1,1,0)[7].

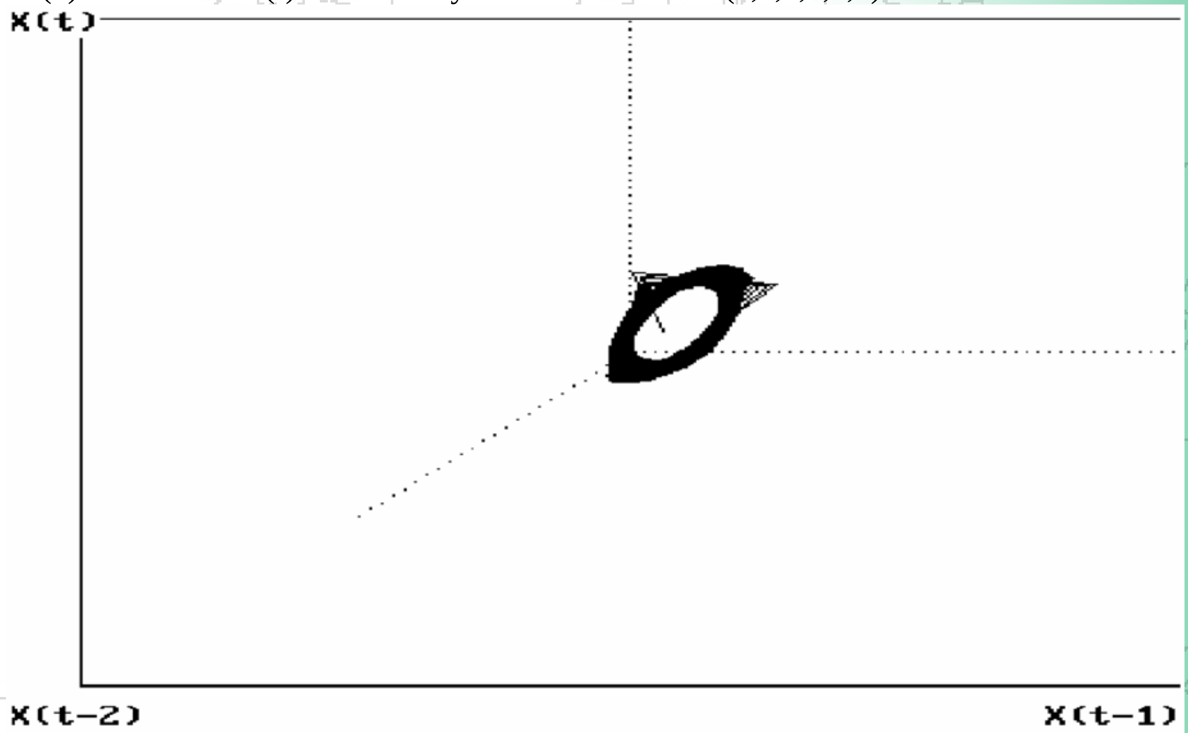


Figure 3. Three-dimensional phase space graph of the trimestral biomass of the yellowfin tuna fish from the Eastern Pacific Ocean, as estimated by the neuronal network model.

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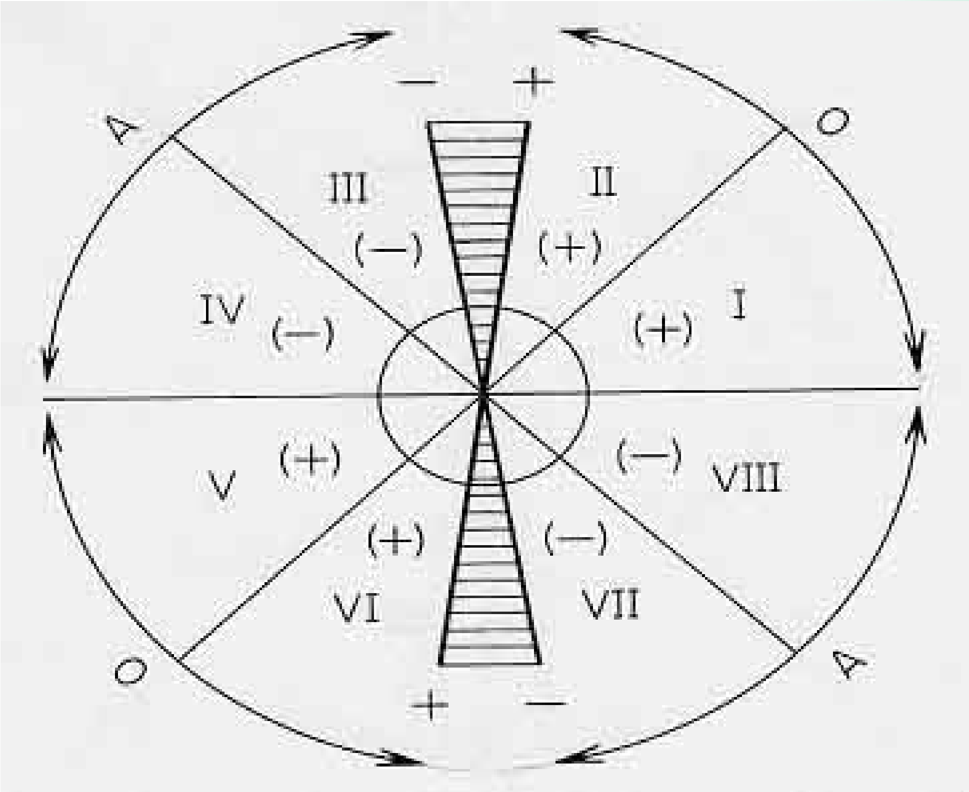


I and V :Stable oscilatory
II and VI :Unstable oscilatory
III and VII :Unstable asintotic
IV and VIII :Stable asintotic

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O : Oscilatory
A : Asintotic

figure 4 Graphic method modified by Ritter et al (2004) in order to calculate phase space eigenvalues as well as the system's behavioural patterns divided in a: oscillatory, asymptotic, stable or unstable and chaotic area.


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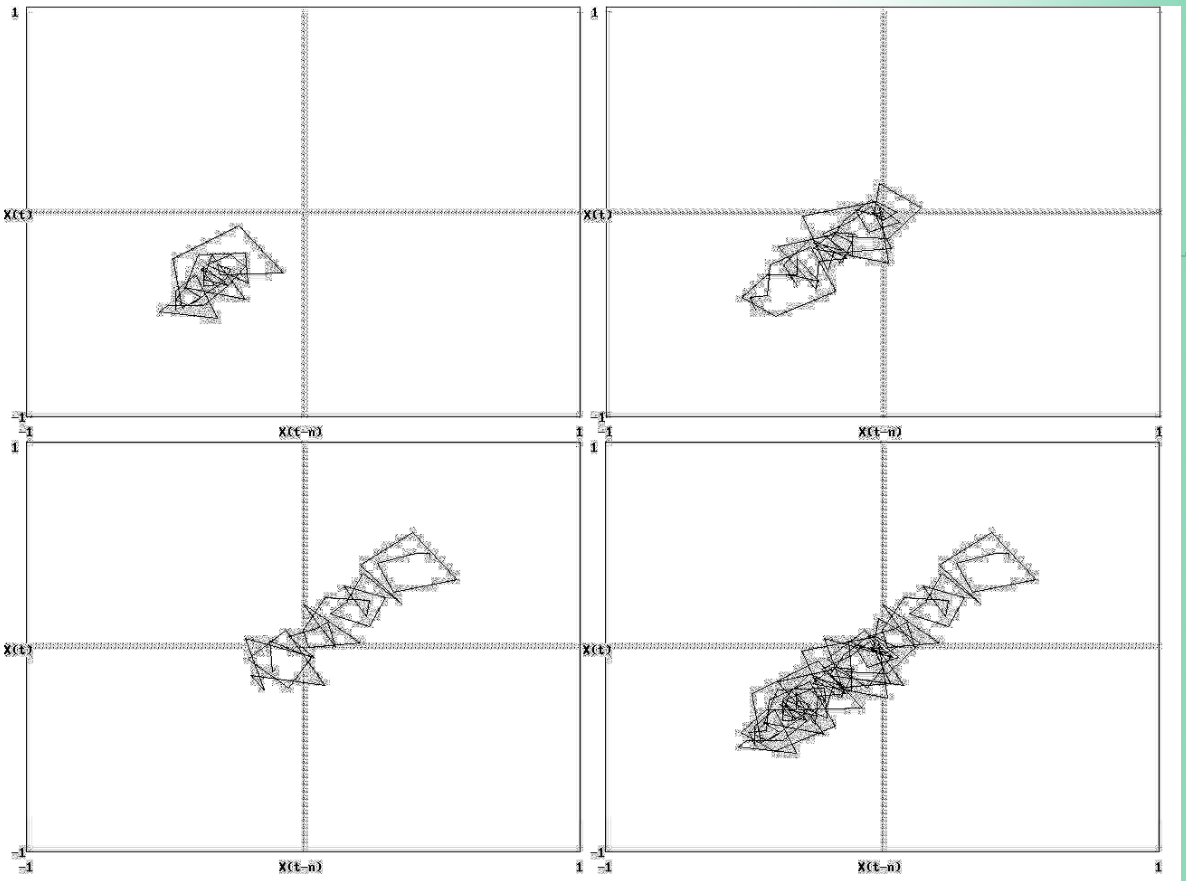


Figure. 5 Global temperature anomalies space phases showing in separate form its multiple attractors and then integration.

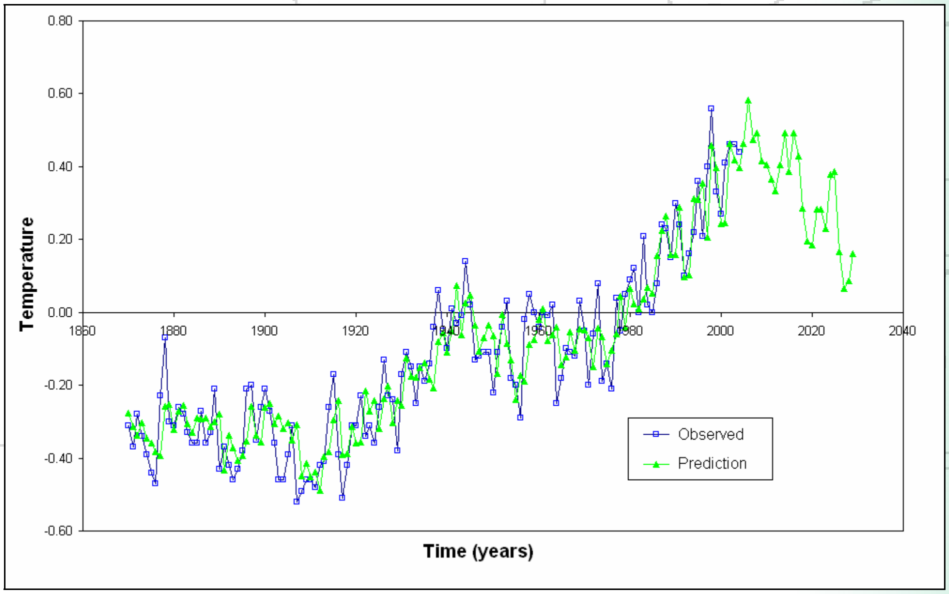


Figure 6 Simulation and prediction of global temperature anomalies using ARIMA statistical method.

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