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TUBES
Generation of novelty in a discrete time system with an unique starting element

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Abstract : This is a journey in complex systems. Starting with a tube made of a "virtual material" and applying a set of couples of operators - cut/paste, fold/unfold, pierce/patch - at each clock tic, systems more and more complex appear at each generation.
The research has the ambition to model such innovative shapes generation and try to provide a new look into the complex systems generation from simpler shapes, using simple operators.
The research is pursued within the POLIS initiative at the Institute of Applied Mathematics of the UCO.

Résumé : C'est un voyage dans le monde des systèmes complexes. En commençant par un tube réalisé dans un « matériel virtuel » et en appliquant un ensemble de couples d'opérateurs – couper/coller, plier/déplier, percer/rustine – à chaque « tic » d'horloge des systèmes de plus en plus complexes apparaissent à chaque génération.
La recherche a l'ambition de modéliser cette génération de formes nouvelles et de proposer une nouvelle vision de la génération de systèmes complexes à partir de formes simples, en utilisant des opérateurs simples.
Cette recherche se déroule dans le cadre de l'initiative POLIS à l'Institut de mathématiques Appliquées de l'UCO.

When my son was about three years, he told me : "people have in their heads small tubes". I decided to have a closer look to this "tubes" structure, at a first glance quite commonplace. Later I took it as a starting point for the study which follows.

The tube is a simple structure, which - by application of certain operators described later – could change and generate complex structures right from the first tenth of sequential applications.
To evacuate from the start the issues related to the tube material, we invent a special material: the "hyper-elast", material which is in a permanent contraction state, and will always find th way to maintain itself at a minimal surface (and with minimal external edges) whatever the operation performed on the material (on the object built out of the material). It will also have the property to reduce the created volumes at a minimum. Later on, we will see in action this very practical material for our purpose.

Sure, some of the properties might look extravagant, but for revealing of novelty in the system, these are rather useful. Actually, this approach allows, as the research progresses, to relax some of these properties and to come closer to the reality by successive steps.

To move in time from one generation to another, a discrete time materialized by a clock tic synchronized with each operator application, will be used.
The processing of a first case will allow to learn more about the material and the operators.
We will use first the following operators:

- **PASTE** - noted /**C**/ - by applying the operator once, it could stick together edges ; any edge on itself or on another edge (eg: close one end of a tube, or paste the two ends of a tube together to form a torus). This operator can work when the object at hand has at least an edge (it doesn't work on a closed, edgeless object like a sphere or a torus).

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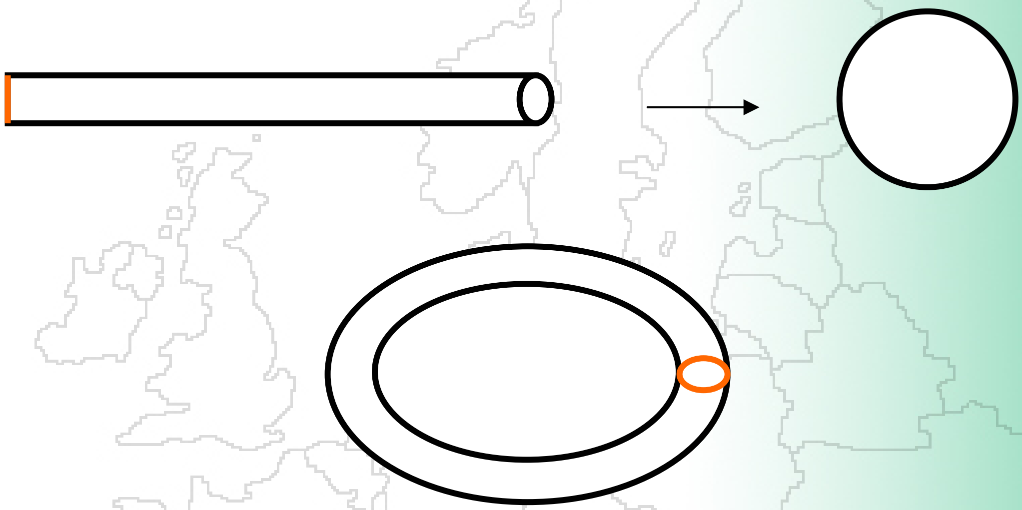
- **CUT** - noted /**K**/ - by applying the operator once, it performs a single cut on a single object, cut which either starts at one edge to join another, or starts at a point to join it again after having done a curve without crossing another edge (eg : transverse cut of a tube in two pieces ; longitudinal cut from one edge to the other to obtain a unique surface ; cut a hole in a tube).

A few simple rules to perform the process:

- at each clock tic one single operator (/**C**/ or/**K**/) is applied only once
- the operator is applied on a single object

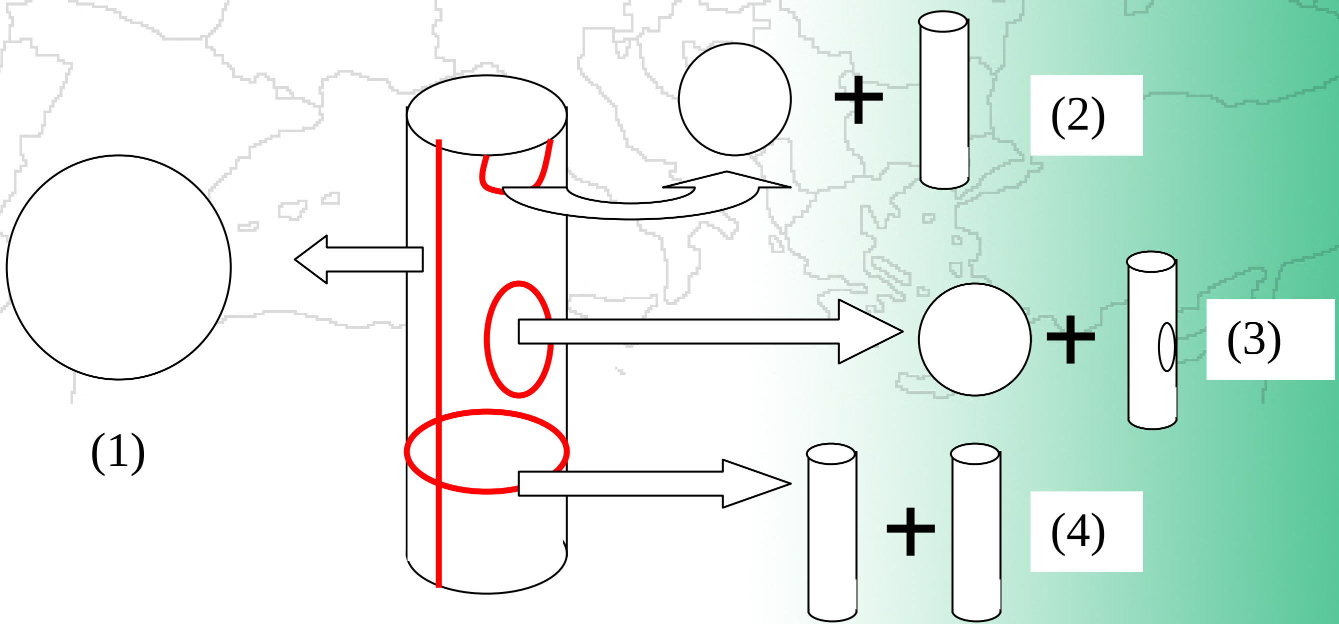
The root of the system is a tube noted « **I** », built out of the material described above. It is an empty cylinder open at the two ends. By applying on **I**, at t=0, one of the two operators, the result at t=1 (generation **G1**) is:

➢ **/C/->I => D or T** (**D**=disk; **T**=Torus)



By application on the same root of the operator **/K/** :

/K/->I => D or [D,I] ou [D,I1] ou [I,I]



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(Remark: at the limit (a cut tangent to one edge of the tube) the situation is not clear : one could consider the result as being **[I,D]** or **[I1,D]** or **[D,D]**, results which we will not take in account for the time being).

The "or" in the cases described above is exclusive, in the sense that we can have at time « t » only one single result. This choice might be left at random or defined using specific rules. Nevertheless for what we should show in the following paragraph, the issue is not relevant.
Let us consider the root **I** t the generation **G0; D,T,[I,I],[D,I]** and **[I,I1]** are the potential elements of the first generation **G1** (for the time being we discard the special cases remarked above).
By applying **/C/** or **/K/** on the objects of generation **G1**, the generation **G2** may be obtained:

- **/C/->D => S (sphere)**
- **/C/->T => T ("paste" on a torus is inoperant: it is edgeless)**
- **/C/->II => [D,I] or [I,T]**
- **/C/->[D,I] => [S,I] or [D,T] or [D,D]**
- **/C/->[D,I1] => [S,I1] or [D,T1] (I1 – tube with a hole/ T1 – torus with a hole)**

- **/K/->D => [D,D] or [D,I]**
- **/K/->T => I or [D, T1]**
- **/K/->II => III or [D,I] or [D,I,I1]**
- **/K/->[D,I] => [D,D,I] or [D,D] or [D,I,I] or [D,D,I1]**
- **/K/->[D,I1] => [D,D,I1] or [D,D] or [D,I,I] or [D,I,I1] or [D,D,I2]**

Let us also further simplify this example by ignoring all the multiple objects which do not contain novelty from one generation to the other and by keeping only the combinations which bring novelty :
G0 -> I
G1 -> D, T, [D,I1]
G2-> [S,I], [D,T1], [D,D,I2],
and extract from such selected groups only novelty:
G0 ->I
G1 -> D, T, I1
G2 -> S, T1, I2
In **G3,** by applying **/C/** on the novelties of **G2,** the novelty **T2 (** a torus with two holes, from **I2)** might be obtained, **/K/** on the same objects of **G2,** the novelty **I3** should be obtained**.**
G3 -> T2, I3
By performing the same operation on **G3,** one would obtain:
G4 -> 2L (2L, a « pair of glases »(a torus with two empties) **–** by application of **/C/** on **T2), T3, I4**
G5 -> T4, I5, 2L1
G6 -> T5, I6, 2L2
G7 -> T6, I7, 2L3, 3L (3L, a torus with three empties**) –** by application of **/C/** on **2L2)**
…
Some bulk observations:

- If **/K/** is applied on **D** or on **S** , **[D,D]** might be obtained**.** This means that **[D,D]** is the result either of **/K/->S** or of **/K/->D** , and therefore an indeterminacy is introduced in the possibility of knowledge of the past (if faced with **[D,D]** without more information).

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- Applying **/C/** to completely closed structures is inoperant (spheres, tori , "glases"...) ; by the contrary **/K/** might always be applied whatever the structure, with different results obtained.

- The »glases » (with 2,3,4... empties) show up for the first time respectively at **G4, G7, G10...** from **T2, 2L2, 3L2… (**which appear themselves at **G3, G6,G9...)** and to which **/C/** is applied.
- The same structure might show up at different generations from different operators sequences.
- The « glases » configurations with many empties (the empties are in the sense of the empty of a torus **T**) are not simple to imagine. There are cases where a 3rd dimension cannot be avoided. The visualization becomes difficult.

This leads us to the observation that a high level of complexity could be achieved with a couple of rules and a simple root. At this point, it is also possible to affirm that novelty is generated systematically, and that the root and the structures revealed at a point of the experiment are reproduced later. Once a novelty reveals, it will show up again later.

The principle being understood by now, and the mechanisms of the process in place, it becomes possible to imagine the building of a machine which would be capable of creating automatically new objects and present these graphically.
Other objects than « **I** » could be the root for the process.
Other couples of operators could be envisaged too: punch/patch fold/unfold,… but these go beyond the purpose of this paper. But these other choices do not change the way novelty reveals.

Having "**S**" as root at **G0**, **S, [D,D]** may be obtained at **G1**
/C/->S =>S
/K/->S => [D,D]
At **G2** the new element **I** is obtained by applying the operators to **D** (a new element of **G1)**.
/C/->D =>S
/K/->D =>[D,D], [D,I]

In this case, **G2**, for the object **I**, is equivalent to **G0** from the first example, **I** being a new novelty root and knowing that **S** and **D** do not produce other novelty.
To summarize: in **G0** : **S,** in **G1** : **D**, in **G2** : **I**.
By having **D** as root : in **G0** : **D,** in **G1** : **S, I.**

In this case, **G1** for the object **I** is equivalent to **G0** of the first example, as explained earlier. What was presented up to now allows to understand the general approach of this paper.
Time has come to make a further step into complexity by giving a new degree of freedom to the process unwinding, and allowing the possibility of the /**C**/ operator application to join two different objects of the same group (ie: outcome of one /**K**/ operation done at the previous generation)

By application of /**K**/ on the root **I**, we obtain four object groups:
- **/K/->I => D or [D,I] or [D,I1] or [I,I]**

By application of /**C**/ on the objects of the **G1** generation, it is possible to obtain at **G2**:
- /**C**/-> **D => S**
- /**C**/-> **[I,I]** => **[I,T]**
- /**C**/-> **[D,I] => [S,I] or [D,T] or [D,D]**
- /**C**/-> **[D,I1] => [S,I1] or [D,T1]**

and more, a new series issued by relaxing the rule:
- **/C/ -> [I,I] => I**

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- **/C/ -> [D,I] => D**
- **/C/ -> [D,I1] => I**

No novelty production in this specific process.
Would it be possible to produce novelty by associating the objects of the same generation, whatever the group they're belonging to?
The reply is the following:
By selecting the two objects **T1** coming from:
/K/->T => I or [D, T1]
/C/->[D,I1] => [S,I1] or [D,T1]
and by applying /**C**/ to the couple **T1-T1**, one obtains **2L.**

This means that the novelty **2L** shows up one generation earlier (in **G3** instead of **G4**), and consequently, if rules are relaxed, novelty could appear earlier, knowing that it is necessary to have a population containing elements from which novelty can be produced. Actually, no novelty was produced, but just a time difference in production of the novelty.
It is also useful to observe that in this case the production of novelty is rather rare. If considering the full set of operations which lead from **G1** to **G2**,

- **/C/-> D => S**
- **/C/-> T => T**
- **/C/-> [I,I] => [I,T] or I**
- **/C/-> [D,I] => [S,I] or [D,T] or [D,D] or D**
- **/C/-> [D,I1] => [S,I1] or [D,T1] or I**

and/or

- **/K/->D => [D,D] or [D,I]**
- **/K/->T => I or [D, T1] (T1 is a torus with a hole)**
- **/K/->II => III or [D,I] or [D,I,I1]**
- **/K/->[D,I] => [D,D,I] or [D,D] or [D,I,I] or [D,D,I1]**
- **/K/->[D,I1] => [D,D,I1] or [D,D] or [D,I,I] or [D,I,I1] or [D,D,I2],**

we know that it is not possible to have the whole setat the same time.

An operator choice has to be made to produce the **G3,** from all the different sets ranging from **{/C/,/C/,/C/,/C/,/C/}** to **{/K/,/K/,/K/,/K/,/K/} -** 2^5=32 combinations are possible.
For example, the operators set **{/C/,/K/,/C/,/K/,/C/}** applied respectively on **{D,T,[I,I],[D,I],[D,I1]},** result in **{S, I or [D,T1], I or [I,T], [D,D,I] or [D,D] or [D,I,I] or [D,D,I1], [S,I1] or [D,T1] or I}.** There is here as well, a choice between 48 possibilities.
Assuming an equiprobability for each combination, which means 3x2^9 possibilities, even if the combination **T1,T1** appears in more than one case, it remains rare.

As a summary, novelty is rather rare, and it becomes even more, as generations accumulate. In certain cases, novelty show up could be forecasted, but unfortunately, as it is novelty, it cannot be known in advance. The operation has to be performed to describe the outcome. From there on it seems difficult, with the today computer equipment, notably the software available, to produce a machine which creates novelty and represents it. This might be feasible nevertheless in the future.
A phenomenon appearing at a higher generation number is the following.

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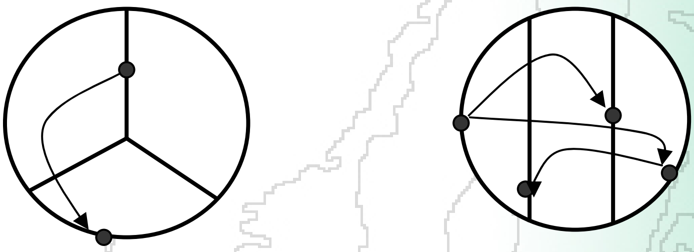
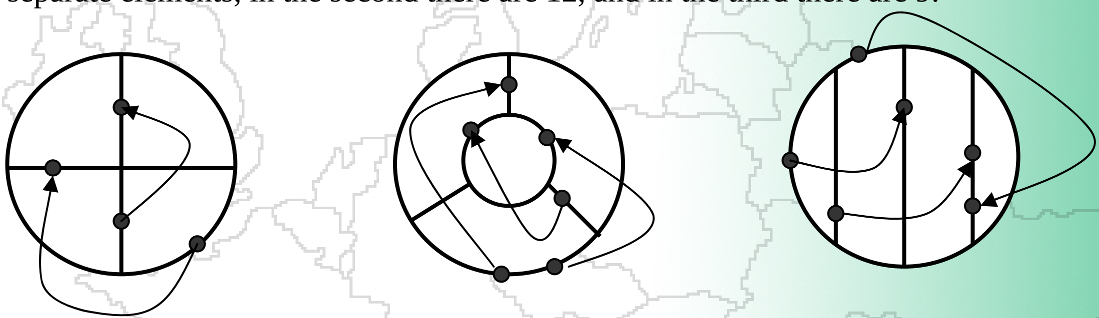
If considering closed forms with three "empties", with two holes placed in different regions of the object, regions which are not adjacent, to apply /C/ it is mandatory to pass over (under) the element which separates the holes.

There are 3 possibilities in the first structure as well as in the second one.

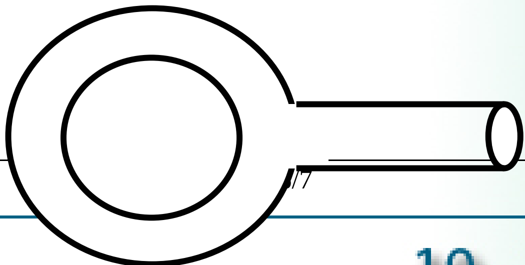


A shape with four empties might have different structures and more possibilities to have holes separated by one or more neighbor elements.
In the cases underneath, from left to right, in the first structure there are 10 different possibilities to join the separate elements, in the second there are 12, and in the third there are 9.



This way leads to a development in real 3D, which wasn't the case for the generations at the beginning, where the 3rd dimension was not necessary to describe the object. From generation **G9**, real 3D structures are observed. Sure there also is a combinatorial explosion of possibilities as generations progress. The objects which appear are close to knots and the complexity goes on limitless.
By adding other couples of operators (as those mentioned earlier, actually fold/unfold and pierce/patch), the system might rapidly arrive at a complexity close to that of proteins.
If creating an environment which is favorable to the manifestation of novelty, a sort of « soup » containing operators in which root elements are plunged, the new elements might appear faster and in larger numbers as well.

It would also be interesting to come back on the initial hypothesis related to the material which is "fully retractable". It is illustrated by the fact that if /**C**/ is applied to stick together a **T1** and an **I** shapes the result will be **T1**, **I** being somewhat absorbed by **T1** (same happens if /**C**/ is applied on **I1** if one edge of the tube is glued to the tube center hole).
If this wouldn't be the case (change in material's characteristics) another shape appears as showed :



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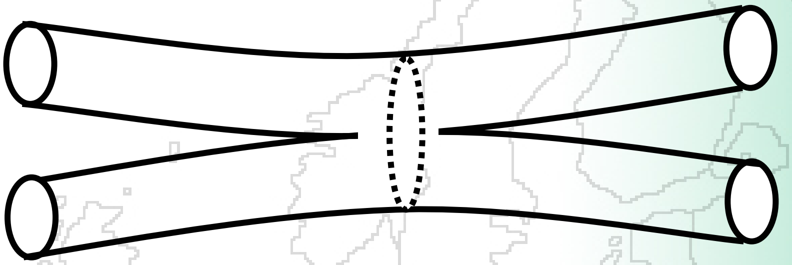
Here we enter a situation where the discussion is no more only topological, but the importance of dimension is brought to the picture.

In the same vein, another interesting case is that of pasting two **I1** on their central hole and then, cutting it in the middle. Two "trousers" appear.



The whole set of cases seen in the previous few paragraphs are novelty producing and bringing complexity as well. Again, if we relax the properties (in this later case, of the material) we come closer to reality and ipso facto augment the complexity.

As a conclusion, we might affirm that the model proposed in this paper is of interest in studying the way in which complexity appears in a structure with development rules. Such model, which remains to be further refined, might lead to a better understanding of structure per se, to a better understanding of the significance of operators as fold/unfold, so important in proteins, and maybe could lead to a novelty producing machinery. The field is close to virgin and a lot of research should be undertaken to bring further what is here a modest insight in the complexity world.