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QUANTUM GRAVITY IN THE SOLAR SYSTEM

We present a study of the solar system based on quantum gravity QG . The nine planets of the solar systems describe elliptic orbits around the sun and each orbit defines the average distance from the sun , e.g. 149.6 millions Km for Earth , 57.9 million Km for Mercury , 5914 millions Km. for Pluto and so on. We show that such quantities are not arbitrary nor random, but they are determined by a general stability condition that produces quantum effects similar to those in hydrogen (QED). The fundamental stable orbit is that of Mercury $p_{Mercury}$ (the correspondent of the Bohr orbit with radius r_B) while the remaining orbits can be related to $n^2 p_{Mercury}$. In particular $n = 1, 1.618, 2, 5, 7, 8, 10$ for Mercury , Earth , Mars, Saturn, Uranus, Neptune , Pluto. The orbits of Jupiter and Venus are directly bonded to the orbit of Saturn. Self-rotation of the sun is the primary cause of quantization .

1. INTRODUCTION

We present a study of the solar system based on quantum gravity. Quantum gravity QG is a not yet satisfactory defined theory whose object, in very crude terms, is to define a gravitational correspondent of quantum electrodynamics , so as to hopefully arrive at a unified formulation of the basic laws of universe . A variety of attempts in such direction are reported in literature , among which those making reference to *string theory* seem at present time the most popular. However , these attempts do not lead to relevant results for practical purposes and certainly they have not afforded a crucial problem: is it possible to explain the orbits of the planets of the solar system according to a defined quantization rule ? The answer to this question represents the object of our study. We try to illustrate the aforesaid correspondence between quantum gravity and quantum electrodynamics , by expounding quantum behaviour as a deterministic behaviour determined by the condition that the observed orbits , electric or gravitational , be *stable*. The word *stability* could probably discourage the reader since *stability theory* is an extremely complex theory and only few contributions (above all by Poincarè and Prigogine) are reported in the literature on classical physics. We soon say that we make reference to an extremely simplified formulation which , nevertheless, proves to be exceptionally powerful . We reduce the stability problem to the problem of determination of minima and maxima of the *total potential energy* E_p . This quantity is conveniently defined and represents in fact the sum of three distinct contributions. We start with QED: in hydrogen the *admissible* orbits of the revolving electron are quantum orbits identified by the Bohr radius r_B and by $n^2 r_B$, where $n = 2, 3, 4$ etc. It has been shown [1] [2][3] that these orbits are stable for n odd and unstable for n even . The *total potential energy* is the sum of the classical Coulomb energy , of the energy relating to the centrifugal force and of the energy arising from the rotation of the electron in the magnetic field generated by the proton. Why does the proton generate a magnetic field? Simply because of its dynamic structure . In fact the proton consists of three quarks in synchronous rotation around their center of mass. Also , the energy relating to the centrifugal force accounts for the fact that in a classical two-body problem , electrical or gravitational, the acting force (of Coulomb or of Newton) is counterbalanced by a reaction centrifugal force proportional to (v^2 / R) . As a result the total potential energy is a damped oscillatory function of R , i.e. of the distance of the electron from the center of mass of the proton. The *extrema* of such function determine the stable and unstable solutions , which correspond to the Bohr radius (basic stable orbit) and to $n^2 r_B$.

In gravitation, things are similar : with reference to the solar system the sun represents the correspondent of the proton and each of the nine planets is the correspondent of the electron (with obvious differences that we try to put into evidence, first of all each planet is different from the others and moreover the various orbits are not contained in the same plane).

As the proton is a rotating body , in the same way the sun is a rotating body and the rotation generates a gravi-magnetic field that influences the motion of each planet. Galileo Galilei with his celebrated observations first produced evidence of the rotation of the sun . Of course this phenomenon is fully confirmed by observations at present time . However a clear theoretical explanation of the phenomenon has not been given yet. We prove that the following is satisfactory : the couple sun- Jupiter defines a two-body system in which both bodies rotate round their common center of mass and the rotation radii are inversely proportional to the masses. As the mass of the sun is about one thousands times the mass of Jupiter, the radius of the orbit of the sun is about one thousands time smaller than the radius of the orbit of Jupiter and coincides , in practice , with the physical radius of the sun ($6.97 \cdot 10^8 Km$). Therefore the sun is at rest with respect to the center of mass , which falls inside the center of the sun itself, but rotates around it . In parallel Jupiter revolves according to the usual law. The self-rotation of the sun has a period of about 25,5 days and does not represent at all an arbitrary phenomenon , disjointed from the physical coupling of the sun with the planets (first of all Jupiter) . This fact has deep implications : the distribution of the planetary orbits is not determined by the Newton 's law only but by the gravi-magnetic field generated by the self rotation of the sun , as well.

2. SELF ROTATION OF THE SUN

It is

$$\frac{M_{sun}}{M_{Jupiter}} = 1047 \quad (2.1)$$

$$R_{Jupiter} = 778.3 \cdot 10^6 Km \quad (2.2)$$

The radius of the massive sun is given by

$$R_{sun} = 6.97 \cdot 10^8 Km \quad (2.3)$$

Consequently the following relation turns out satisfied

$$(16/15)R_{sun} \cdot M_{sun} = M_{Jupiter} \cdot R_{Jupiter} \quad (2.4)$$

This means that $(16/15)R_{sun}$ is the equivalent revolution radius of the sun in the two-body problem sun-Jupiter. Conversely the radius of the sun is $(15/16)$ the theoretical revolution radius (of the sun) in such problem. Essentially, revolution is transformed into self-rotation . The factor $(15/16)$ can be explained as an effect of the non-uniform distribution of the mass of the sun. The mass is concentrated within a sphere of radius $0.25 R_{sun}$ and then

$$\pi R^2 - \pi \cdot [0.25R]^2 = \pi R^2 \frac{15}{16} \quad (2.5)$$

$$\frac{\pi R^2 - \pi \cdot [0.25R]^2}{\pi R^2} = \frac{15}{16} \quad (2.6)$$

Also, in view of the determination of the moment of inertia we replace M_{sun} with M_{sun}^* ,

$$M_{sun}^* = M_{sun} \times \frac{(4/3)\pi(0.25R_{sun})^3}{\pi R_{sun}^2 \cdot 0.25R_{sun} \cdot (15/16)} \quad (2.7)$$

$$= M_{sun} \times \frac{5}{64}$$

and , in parallel we set

$$I_S = M_{sun} \cdot R_{sun} = M_{sun}^* \cdot R_{sun}^* \quad (2.8)$$

It follows from (2.7) (2.8) that

$$R_{sun}^* = 3.577 R_{sun} \quad (2.9)$$

The equation of the rotation of the sun is given by

$$\dot{\omega} = \omega^2 - \frac{GM_{Jupiter}}{(R_{sun}^*)^3} \quad (2.10)$$

with G the gravitation constant.

The condition of equilibrium is defined by

$$\dot{\omega} = 0 \quad (2.11)$$

and implicates that

$$\omega^2 = \frac{GM_{Jupiter}}{(R_{sun}^*)^3} \quad (2.12)$$

As $M_{Jupiter} = 1.898 \cdot 10^{27} \text{ Kg}$, $R_{sun}^* = 3.577 \times 6.97 \cdot 10^8 \text{ m}$, $G = 6.67 \cdot 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2$

it comes out

$$\omega = 2.858 \text{ rad} / \text{s} \quad (2.13)$$

Consequently the rotation period is

$$T_{sun} = \frac{2\pi}{\omega} = 2.198 \cdot 10^6 \text{ s} \approx 25.52 \text{ days} \quad (2.14)$$

In substantial agreement with observations. As regards eq. (2.10) note that it can be derived from the eq. of the revolution of the sun round the center of mass sun-Jupiter through the following steps:

$$(s_1) \quad M_{sun}^* \cdot \ddot{R}_{sun}^* = M_{sun}^* \cdot \frac{v_{sun}^2}{R_{sun}^*} - \frac{GM_{Jupiter}M_{sun}^*}{(R_{sun}^*)^2} \quad (2.15)$$

(s₂) replace \ddot{R}_{sun}^* with $R_{sun}^* \dot{\omega}$; then

$$M_{sun}^* R_{sun}^* \dot{\omega} = M_{sun}^* \cdot \frac{v_{sun}^2}{R_{sun}^*} - \frac{GM_{Jupiter}M_{sun}^*}{(R_{sun}^*)^2} \quad (2.16)$$

It follows from (2.16) that

$$\dot{\omega} = \left(\frac{v_{sun}}{R_{sun}^*} \right)^2 - \frac{GM_{Jupiter}}{(R_{sun}^*)^3} \quad (2.17)$$

(s₃) replace $v_{sun} = \omega R_{sun}^*$ in (2.17).

Then eq. (2.17) gives eq. (2.10).

3. THE BASIC QUANTUM RADIUS (Mercury)

The self-rotation of the sun essentially is a rotation of the external layer, formed by chromosphere + photosphere, with respect to the central part at rest. The thickness Δr of such layer can be estimated about 2555 Km (fig. 1). The corresponding volume is

$$\text{Volume of sun layer} = (4\pi/3)[R_1^3 - R_2^3] \quad (3.1)$$

with

$$R_1 = 6.97 \cdot 10^5 \text{ Km} ; R_2 = 6.97 \cdot 10^5 \text{ Km} - 2555 \text{ Km} = 6.9444 \cdot 10^5 \text{ Km}$$

$$R_1^3 - R_2^3 = 3.7095 \cdot 10^{15} \text{ Km}^3$$

Such volume is equivalent to that of a cylinder with basis πR_1^2 and height $2y$, with

$$y = \frac{(4\pi/3)[R_1^3 - R_2^3]}{\pi R_1^2} = 1.0184 \cdot 10^4 \text{ Km} \quad (3.2)$$

Setting

$$\varphi = 45^\circ - [(\psi_{rot})_{mercury} + (\psi_{rot})_{sun}] = 45^\circ - (28^\circ + 0.7^\circ) \quad (3.3)$$

with

$(\psi_{rot})_{mercury} = 28^\circ$ angle of the rotation axis of mercury with respect to the vertical to the orbit plane

$(\psi_{rot})_{sun} = 0.07^\circ$ angle of the rotation axis of the sun with respect to the vertical to the equatorial plane

we find

$$\tan \varphi = 0.304 \quad (3.4)$$

From (3.2) and (3.4) we gather the relation (see fig. 2)

$$y \cdot \tan \varphi = 3100 \text{ Km} \quad (3.5)$$

From (3.5) we derive the average radius $p_{mercury}$ through the eq.

$$p_{mercury} = (y \cdot \tan \varphi) \times (16\pi e)^2 \quad (3.6)$$

in which e is the Neper constant. Eq. (3.6) together with (3.5) gives

$$p_{mercury} = 57.9 \cdot 10^6 \text{ Km}$$

in agreement with experimental findings .Of course it is understood the orbit of mercury is an ellipsis with radius

$$R_{mercury} = \frac{p_{mercury}}{1 + \varepsilon \cos \theta}$$

where ε is the eccentricity.

We do not illustrate the derivation of eq. (3.6) but we emphasize that this eq. is the gravitational correspondent of the following electric relation which determines the Bohr radius in hydrogen:

$$r_B = y \cdot \tan 45^\circ \times (16\pi e)^2$$

with

$$y = 2 \times 1.41 \cdot 10^{-15} \text{ m} = 2 \times \text{average radius of the proton}$$

In conclusion , as in hydrogen the rotation of the proton (which in reality is the joined effect of the synchronous rotations of the three constituent quarks) determines a quantization of the admissible orbits of the revolving electron (so that the basic stable orbit is that of Bohr), in a similar way the self-rotation of the sun determines the fundamental stable orbit, among those covered by the planets of the solar system , i.e. the orbit of Mercury. On the other hand while the proton and the electron have bi-dimensional structures which, moreover, are complanate , the sun and Mercury (as well as the other planets) have three-dimensional structures and the angle between their rotation axis is different from zero.

Note It is $(16\pi e) = 136.63 \approx 1/\alpha$ with α the fine structure constant.

4. THE ORBIT OF MARS

The average distance of Mars from the sun is

$$p_{Mars} = 228.8 \cdot 10^6 \text{ Km} \quad (4.1)$$

and the orbit of Mars is not complanate with that of Earth nor with that of Mercury. We now show that the orbit of Mars is bonded to the orbit of Mercury, i.e. to the fundamental stable orbit, by the following quantum relation

$$p_{Mars} = p_{Mercury} \times \left[n \cos(\theta_{Mars} + \theta_{Mercury}) \right]^2 \quad (4.2)$$

$n=2$

where θ_{Mars} is the angle formed by the orbit of Mars with the plane of the orbit of Jupiter and $\theta_{mercury}$ is the angle formed by the orbit of Mercury with the plane of the orbit of Jupiter :

$$\theta_{mars} = 0.545^{\circ} ; \theta_{Mercury} = 5.699^{\circ} \quad (4.3)$$

In fact as

$$\begin{aligned} \cos(\theta_{Mars} + \theta_{Mercury}) &= \cos 6.244^{\circ} = 0.994 \\ [2 \cos(\theta_{Mars} + \theta_{Mercury})]^2 &= 3.952 \end{aligned}$$

then

$$p_{Mercury} \times [2 \cos(\theta_{Mars} + \theta_{Mercury})]^2 = 57.9 \cdot 10^6 \text{ Km} \times 3.952 = 228.8 \cdot 10^6 \text{ Km} = p_{Mars}$$

Remark

The values of the angles θ_{Mars} and $\theta_{Mercury}$ in eq. (4.3) are directly referred to the plane of the orbit of Jupiter. They can be derived from the usual values referred to Earth, by convenient and obvious operations. The standard values referred to Earth are

$$\theta^E_{Mercury} = 7.005^{\circ}; \theta^E_{Mars} = 1.850^{\circ}; \theta^E_{Jupiter} = 1.305^{\circ}$$

The same criterion is adopted for determining the angles of the remaining planets (e.g. see Saturn in following Sec. 5) . In conclusion, according to our scheme the orbit of orbit is a quantum orbit with $n=2$.

5.THE ORBIT OF SATURN

The average distance of Saturn from the sun is $p_{Saturn} = 1426.6 \cdot 10^6 \text{ Km}$. The angle of the orbit of Saturn with respect to Earth is $\theta^E_{saturn} = 2.488^{\circ}$ and then the angle referred to Jupiter is

$$\theta_{Saturn} = \theta^E_{Saturn} - \theta^E_{Jupiter} = 2.488 - 1.305 = 1.831^{\circ}$$

It comes out

$$p_{Saturn} = p_{Mercury} \times \left[n \cos(\theta_{Saturn} + \theta_{Mercury}) \right]^2$$

with

$$n = 5$$

In fact

$$\begin{aligned} \cos(\theta_{Saturn} + \theta_{Mercury}) &= \cos 6.822^{\circ} = 0.9928 \\ [5 \cdot \cos(\theta_{Saturn} + \theta_{Mercury})]^2 &= 4.964^2 = 24.641 \end{aligned}$$

$$p_{Mercury} \times [5 \cdot \cos(\theta_{Saturn} + \theta_{Mercury})]^2 = 57.9 \cdot 10^6 \text{ Km} \times 24.641 = 1426.6 \cdot 10^6 \text{ Km} = p_{Saturn}$$

In conclusion : according to our scheme the orbit of Saturn is a quantum orbit with quantum number $n=5$

6. THE EXTERNAL PLANETS (Uranus, Neptune, Pluto)

The external planets , Uranus, Neptune, Pluto , have average distances from the sun

$$p_{Uranus} = 2870 \cdot 10^6 \text{ Km} ; p_{Neptune} = 4495 \cdot 10^6 \text{ Km} ; p_{Pluto} = 5813 \cdot 10^6 \text{ Km}$$

They rotate in planes having the following angles with the Earth plane

$$\theta^E_{Uranus} = 0.77 \text{ rad} ; \theta^E_{Neptune} = 1.77^0 ; \theta^E_{Pluto} = 17.14^0$$

Moreover the rotation of Uranus and that of Pluto are *retrograde* , which means that the verse of the rotation is opposite to the verse of the revolution.

We find that the quantum formula for Uranus is

$$p_{Uranus} = p_{Mercury} \times \left[\frac{n}{\cos(-\theta_{Uranus} + \theta_{Mercury})} \right]^2$$

with

$$n = 7$$

$$\theta_{Uranus} = \theta^E_{Uranus} - \theta^E_{Jupiter} = 0.77 - 1.305 = -0.535^0$$

In fact it is

$$-\theta_{Uranus} + \theta_{Mercury} = 0.535 + 5.699 = 6.234^0$$

$$\cos(-\theta_{Uranus} + \theta_{Mercury}) = 0.9941$$

$$\left[\frac{n}{\cos(-\theta_{Uranus} + \theta_{Mercury})} \right]^2 = \left[\frac{7}{0.9941} \right]^2 = 49.58$$

$$p_{Mercury} \times \left[\frac{7}{\cos(-\theta_{Uranus} + \theta_{Mercury})} \right]^2 = 57.9 \cdot 10^6 \text{ Km} \times 49.58 = 2870.8 \cdot 10^6 \text{ Km} = p_{Uranus}$$

As regards Neptune we find that the orbit can be related to the quantum number $n=9$, provided that we account for a convenient interaction with Pluto and Uranus. More precisely we find

$$p_{Neptune} = p_{Mercury} \times [n \cos(\theta_{npu} + \theta_{mercury})]^2$$

with

$$n = 9$$

$$\theta_{npu} = \theta_{Neptune} - \theta_{Pluto} - \theta_{Uranus}$$

$$\theta_{Neptune} = \theta^E_{Neptune} - \theta^E_{Jupiter} = 1.77 - 1.305 = 0.465^0$$

$$\theta_{Pluto} = \theta^E_{Pluto} + \theta^E_{Jupiter} = 17.14 + 1.305 = 18.44^0 ; \theta_{Uranus} = -0.535^0 ;$$

With above values it is

$$\theta_{npu} = 0.465 - 18.44 + 0.535 = -17.44^0 ; \theta_{npu} + \theta_{Mercury} = -11.74^0$$

$$\cos(\theta_{npu} + \theta_{Mercury}) = \cos(-11.74^0) = 0.979$$

$$[9 \cdot \cos(\theta_{npu} + \theta_{Mercury})]^2 = 8.811^2 = 77.63$$

$$p_{Mercury} \times [9 \cdot \cos(\theta_{npu} + \theta_{Mercury})]^2 = 57.9 \cdot 10^6 \text{ Km} \times 77.63 = 4495 \cdot 10^6 \text{ Km} = p_{Neptune}$$

So , according to our scheme the orbit of Neptune has quantum number $n=9$.

Regarding Pluto we find the following quantum relation

$$p_{Pluto} = p_{Mercury} \frac{n^2}{\cos(\theta_{npu} + \theta_{Mercury})}$$

with

and , like for Neptune

$$n=10$$

$$\theta_{npu} + \theta_{Mercury} = -11.74^{\circ}$$

In fact it is

$$\frac{n^2}{\cos(\theta_{npu} + \theta_{Mercury})} = \frac{100}{0.979} = 102.14$$

$$P_{Mercury} \times \frac{10^2}{\cos(\theta_{npu} + \theta_{Mercury})} = 57.9 \cdot 10^6 \text{ Km} \times 102.14 = 5914 \cdot 10^6 \text{ Km} = P_{Pluto}$$

This means that , according to our scheme , the orbit of Pluto has quantum number $n=10$.

7. EARTH , VENUS AND JUPITER

Earth :the following fascinating quantum relation binds the orbit of Earth to that of Mercury

$$P_{Earth} \approx P_{Mercury} [1.618 \cos(\theta_{Earth} + \theta_{Mercury})]^2 \quad (7.1)$$

$$1.618 \text{ the golden ratio ; } \theta_{Earth} + \theta_{Mercury} = 7^{\circ}$$

In fact it is

$$1.618 \cos 7^{\circ} = 1.6059^2 = 2.579$$

$$P_{Mercury} \times 2.579 = 57.9 \times 2.579 = 149.32 \cdot 10^6 \text{ Km} \approx P_{Earth} = 149.6 \cdot 10^6 \text{ Km}$$

Hence , according to our scheme the quantum number relating to Earth is $n= 1.618$. Note that such number is not an integer and this particularity is certainly to be connected to the joined influence of Jupiter and Venus. In other words , unlike for Mercury, Mars , Saturn ,Uranus , Neptune and Pluto , the primary quantum effects of the rotation of the sun are modified by secondary effects due to Jupiter and Venus. In turn the orbits of Jupiter and Venus are influenced by the motion of Saturn(see the next development).

Jupiter: the orbit of Jupiter turns out related to the orbit of Saturn by the following mass relation

$$\frac{p_{Jupiter}^2}{p_{Saturn}^2} = \frac{M_{Saturn}}{M_{Jupiter} + M_{Earth} + M_{Venus}} \quad (7.2)$$

$$M_{Saturn} = 95.16 M_{Earth} ; M_{Jupiter} = 317.83 M_{Earth} ; M_{Venus} = 0.815 M_{Earth}$$

In fact it is

$$\frac{M_{Jupiter} + M_{Earth} + M_{Venus}}{M_{Saturn}} = \frac{319.64}{95.16} = 3.590$$

while

$$\frac{p_{Saturn}^2}{p_{Jupiter}^2} = \frac{(9.536 p_{Earth})^2}{(5.203 p_{Earth})^2} = 3.590$$

Note that :

1) As $M_{Earth} + M_{Venus} \ll M_{Jupiter}$ eq. (7.2) essentially implicates a direct relation between Jupiter and Saturn . The relation says that the product of the area of the orbit of Jupiter by the mass of Jupiter equals the product of the area of the orbit of Saturn by the mass of Saturn . In other words the sun represents the “area center of mass “of Jupiter and Saturn . As the orbit of Saturn is identified by the aforementioned quantum law, the ratio $(M_{Jupiter} / M_{Saturn})$ identifies the orbit of Jupiter.

2) As formula (7.2) is accurate, it seems reasonable to conjecture that in the early stages of the life of the solar system the three planets Jupiter, Earth, Venus formed a single planet.

Venus: the following equation relates the orbit of Venus to that of Jupiter

$$\frac{p_{Jupiter}^2}{p_{Venus}^2} = \frac{M_{Earth} + M_{Venus}}{M_{Jupiter}} \cdot \left(\frac{M_{Saturn}}{M_{Earth}} \right)^2 \quad (7.3)$$

In fact

$$\frac{M_{Earth} + M_{Venus}}{M_{Jupiter}} \cdot \left(\frac{M_{Saturn}}{M_{Earth}} \right)^2 = \frac{1 + 0.815}{317.83} \cdot 95.16^2 = 51.712$$

$$\frac{p_{Jupiter}^2}{p_{Venus}^2} = \frac{(5.203 p_{Earth})^2}{(0.7235 p_{Earth})^2} = 51.712$$

Eq. (7.3) has a structure similar to (7.2), as it establishes a tie between two areas (that of the orbit of Jupiter and that of the orbit of Venus in this case). In the r.h.s the sum $(M_{Earth} + M_{Venus})$ seems to indicate that at a certain stage of the solar system evolution Earth and Venus formed a single planet. Moreover, the mass of Saturn, in addition to that of Jupiter, is into evidence. Of course we do not have at our disposal, so far, enough data for reconstructing past history. But our formulas reasonably suggest that after a first stage in which, as said beforehand, Earth, Jupiter and Venus formed a single planet, the couple Earth+Venus moved away from Jupiter due to “some” perturbation. The orbit of this couple coincided with the present orbit of Venus and was determined by the combined effects of Jupiter and Saturn. At a third stage Earth moved away from Venus and took the present position in relation to the sun. Table I summarizes the quantum relations for the 9 planets.

8. CONCLUSIONS

We have shown that the orbits of the planets of the solar system are quantum orbits that can be set into a direct relation with the self-rotation of the sun. Such rotation is due to the coupling sun - Jupiter. There is fundamental planetary orbit which is that of Mercury; the remaining ones are bonded to it by equations that are similar to those relating the electron's orbits to the Bohr orbit, in hydrogen. The similarity between quantum gravity (QG) and quantum electrodynamics (QED) essentially descends from the following common property: as the three quarks of the proton (synchronously) rotate round their center of mass, in the same way the sun layer, formed by chromosphere plus photosphere, rotates round the center of mass of the sun. Moreover, as a rotating charge produces an electromagnetic field, in the same way a rotating mass produces a gravi-magnetic field. This is a postulate of ours and turns out fully confirmed by the preceding experimental analysis. A third point is that a mass moving into a gravi-magnetic field (e.g. Mars into the field generated by the sun) puts into play a potential energy which sums up with the Newton gravitation energy and with the centrifugal energy. As a result the total potential energy $E_p(R)$ is a (damped) oscillatory function of the distance R from the sun. The relative minima and maxima identify the average radii of the orbits of the 9 planets: the integral numbers $n=1, 2, 5, 7, 9, 10$ identify Mercury, Mars, Saturn, Uranus, Neptune, Pluto (fig. 3). They can be looked at as “primary” quantization numbers, induced by the sun alone. The intermediate numbers $n=1.365, 1.618, 3.665$ identify Venus, Earth, Jupiter and can be looked at as “secondary” quantization numbers, induced by the sun plus Saturn. Therefore we have found an operative criterion for defining quantum gravity and for correlating it to quantum electrodynamics. It seems remarkable that the quantum number referring to earth is the *golden ratio*. As well remarkable is the fact that the angles among the various planet

orbits determine the exact value of the quantum relations. Note that in a parallel study , not published as yet , we show a significant extension : the orbit of the moon around Earth is the fundamental quantum orbit referring to Earth , after accounting for the self- rotation of Earth. Last, but not least : since many stars are similar to the sun and , moreover , have planets of the size of Jupiter , our results represent a decisive argument in favour of the cosmological existence of planetary systems similar to the solar system.

TABLE I

PLANET	Quantum number	Primary Agent	Secondary Agent	Distance from sun
<i>Mercury</i>	n=1	Sun		57.9×10^6 Km
<i>Venus</i>	n=1.365	Sun	Saturn , Earth	108.2×10^6 Km
<i>Earth</i>	n =1.618	Sun	Saturn , Venus	149.6×10^6 Km
<i>Mars</i>	n=2	Sun		228.8×10^6 Km
<i>Jupiter</i>	n=3.665	Sun	Saturn	778.3×10^6 Km
<i>Saturn</i>	n=5	Sun		1426.6×10^6 Km
<i>Uranus</i>	n=7	Sun		2870.8×10^6 Km
<i>Neptune</i>	n=9	Sun		4495×10^6 Km
<i>Pluto</i>	n=10	Sun		5914×10^6 Km

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