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A macroscopic spatio-temporal model of the brain

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Abstract

A macroscopic spatio-temporal model of brain dynamics is presented. It is here called as the spatio-temporal response model. The prediction of the spatial brain activity from a known initial one is possible with this model. Its mathematical structure is deduced from the time response model that predicts the time brain activity as a consequence of a stimulus, particularly of a stimulant drug.

Let us difference between the time brain activity, with absence of details about its spatial structure, from the spatio-temporal brain activity, with spatial-density (in brain volume) distribution of its activity. In fact, the time brain activity can be deduced from the spatio-temporal brain activity as the spatial integral over the brain spatial domain at any time.

On the one hand, the time response model permits to compute the dynamics of the time brain activity. The time response model is a non-autonomous first order differential equation whose dynamics depends on the particular stimuli. On the other hand, the spatio-temporal response model permits to compute the dynamics of the spatio-temporal brain activity. It is a non-autonomous reaction-diffusion model, which has been deduced from the response model, considering the following hypotheses:

1. Many stimuli of different kinds can affect the brain activity.
2. The brain activity has not a tonic level; it is substituted by the continuous presence of the stimuli that are always influencing the brain.
3. The spatio-temporal dynamics is obtained by substituting in the brain activity the time dependence by the spatio-temporal dependence.
4. A diffusion term is added to describe the particular spatial dynamics.

The analytical solution of the spatio-temporal response model is obtained for the idealized half-sphere geometry of the brain. The boundary conditions, given by the null flow through the brain walls, cause the arising of some quantum numbers that characterize the spatio-temporal dynamics of the brain. In addition, the initial conditions provide the coefficients of the functions' series that takes part in the analytical solution.