On the measure process between different scales

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Quantum Physics (i)

- Matter is constituted by discrete quanta (classical atoms)
  Fact empirically put in evidence by E. Rutherford

- Light is also discretized into quanta:
  Photoelectric effect discovered by H. Hertz
  Photon hypothesis of A. Einstein (1905)

- A major consequence of these discoveries in 21th century:
  existence of lasers, transistors and computers.

Quantum Physics (ii)

- Stability of classical atoms is not understandable in the framework of classical mechanics and electromagnetism.
- Quantum mechanics was developed in the 1930's to explain this stability by N. Bohr, M. Born, L. De Broglie, W. Heisenberg, E. Shrödinger and many others!
- The notion of what a scientist call "experiment" has been to be re-considered.
- Microscopic quanta as classical atoms or photons are not directly perceptible by our senses (M. Mugur-Schächter, 2008).
- Mathematical formalism: vectors, matrices, *etc.*

Quantum Physics (iii)

- Any possible knowledge for a human observer of a microscopic quantum is founded on the experimental protocols.
- The interaction between a microscopic quantum and the measuring apparatus changesthe observed quantum of Nature.
- An *a priori*, an external description of Nature is not possible at quantum scale.
- Philosophical consequences of this new vision of Nature are in progress: B. D'Espagnat, M. Bitbol, B. Nicolescu.

Fractal Geometry

- Importance of scale invariance

  B. Mandelbrot (1975), L. Nottale (1993).
- "Fractal" property

  for figures that are self-similar whatever the refering scale.

Romanesco broccoli

Fractaquantum Hypothesis (i)

- Proposed at the 5th UES Congress (FD, 2002)
- Founded on two remarks:
  Nature develops scale invariance
  Quantum mechanics is relevant for small scales.
- Notion of "atom" (FD, 2004):
  very similar to the way of vision of Democrite
  and the ancient Greek philosophers (see J. Salem, 1997).
- If we divide an "atom" into two parts, its qualitative properties
  change strongly at least in one of these parts.
- An "atom" can be
  a classical atom, or its nucleus,
  or a molecule, or a micro-organism like a cell,
  or an entire macro-organism as a human being
  or till an entire society, or the entire Universe!

Fractaquantum Hypothesis (ii)

- Fractaquantum hypothesis:
  Formulation of quantum mechanics can be applied to all "atoms" in Nature, whatever their size.

- Measure process in quantum mechanics:
  interaction of two "atoms" of different scales:
  A ℓittle "atom" ℓ is a classical atom
  A big "atom" B is a human observer.

- Two "atoms" ℓ and B of different scales:
  "Atom" ℓ is not directly perceptible to "atom" B.
  A direct interaction between B and ℓ is **not** controlled by B himself.

About scale difference and perception

- Two "atoms" ℓ and B have different scales
  when "atom" ℓ is not directly perceptible to "atom" B.

- The perception, *id est* the consciousness of direct interaction between a little "atom" ℓ and a big one B,
  is neglected when ℓ and B have different scales.

- In consequence, the notion of perception between two "atoms"
  should be more precisely defined in future works.

Adding the mathematical framework (i)

Following the Fractaquantum hypothesis

- The measure process of some characteristic of "atom" ℓ follows the mathematical framework of quantum mechanics.
- The "atom" ℓ is modelized mathematically by a vector also denoted by ℓ in an Hilbert space H of configuration.
- The action of measurement is represented by a self-adjoint operator A.
- This operator A is determined by the macro "atom" B which choose the physical quantity to measure and by the rules of quantification.

Adding the mathematical framework (ii)

- The operator A is determined by the macro "atom" B
- The result of the measure process is an eigenvalue α of this operator A
- The "atom" ℓ is projected onto the eigenspace E_α.
- Born rule: the probability of observing the datum α as a result of the mesurement is the squared norm of the projection of ℓ on the eigenspace E_α.

Large open questions and a first idees of unswer

- How to revisit this classical quantum formalism
  when little and big "atoms" are nonclassical ones?
- This research program is tremendous!
  The phenomenology of possible measurement interactions
  should be reconstructed.
  What is a big "atom" B that can measure some quantities
  on little "atom" ℓ?
  Does the classical framework of quantum mechanics
  operates without any modification?
- A very particular example in this work:
  measurement process associated with voting.
  "Atom" ℓ is a social actor and "atom" B is the entire society.
- Scope of the lecture
  1) Voting process and quantum mechanics.
  2) Range voting procedure (M. Balinski and R. Laraki, 2006)

Example with macroscopic "atoms"

- Macroscopic "atom" B: an entire social structure.
  Social actors of society B: little "atoms" ℓ in our model:

  $$\ell \in \mathrm{B}$$

  The number of such indistinguable individuals is quite important (10^6 to 10^9 typically).
- The democratic life in society B supposes that social responsabilities are taken by elected representants.
- A voting process has the objective to determine one particular social actor among all for accepting social responsabilities.
- This kind of position is supposed to be attractive
  A set Γ of candidates γ among the entire set of "atoms" ℓ is supposed to be given.

The question of the election

- Determine a single "elected" candidate γ_1 among the family Γ thanks to the synthesis of all opinions of different electors ℓ.
- Social objective of society B (macro "atom"): determination of one candidate among others through a social process managed by the entire society.
- This problem is highly hill posed! Pioneering works of J.C. de Borda (1781) and N. Condorcet (1785) "Impossibility theorem" of K. Arrow (1951).
- We restrict here to the "first tour" process. Each elector ℓ transmits the name of at most one candidate γ. An ordered list of candidates is obtained by counting the number of expressed votes for each candidate.

Quantum model for an election process (i)

- Space H_Γ of candidates
  genereted formaly by the finite family Γ of all candidates:

$$H_\Gamma = \bigoplus_{\gamma \in \Gamma} \mathbb{C}\, \gamma$$

- This previous decomposition is supposed to be orthogonal:

$$< \gamma \,|\, \gamma' > = \begin{cases} 0 & \text{if} \quad \gamma \neq \gamma' \\ 1 & \text{if} \quad \gamma = \gamma', \end{cases} \quad , \qquad \gamma, \gamma' \in \Gamma.$$

- The "wave function" associated with an elector ℓ
  is represented by a state denoted by $|\, \ell >$ in this space H_Γ:

$$|\, \ell > = \sum_{\gamma \in \Gamma} < \ell \,|\, \gamma > \; |\, \gamma > \; .$$

Quantum model for an election process (ii)

- Scalar product $<\ell\,|\,\gamma>$ in relation $|\,\ell>=\sum_{\gamma\in\Gamma} <\ell\,|\,\gamma>\,|\,\gamma>$

  Component of elector $|\,\ell>$ relative to each candidate γ.
  Political sympathy of elector ℓ relatively to the candidate γ.
- Norm $\|\,\ell\,\|$ of state $|\,\ell>$:

$$\|\,\ell\,\| \equiv \sqrt{\sum_{\gamma\in\Gamma} |<\ell\,|\,\gamma>|^2}$$

is **inferior or equal** to unity.

- Born rule: the probability for elector ℓ to give its vote to candidate γ is equal to $|<\ell\,|\,\gamma>|^2$.
- Probability to unswer by a vote "blank or null": $1-\,\|\,\ell\,\|^2$.

Violence of the projection process (i)

- Projection process in the quantum measurement
  for such a first tour of election process
- During the particular day where the measure process occurs,
  the elector ℓ is obliged to choose at most one candidate γ_0.
- All his political sensibility is socially "reduced"
  to this particular candidate: $|\,\ell > = \; |\gamma_0 >$
  to express the wave function collapse.

Violence of the projection process (ii)

- No elector has political opinions that are identical to one precise candidate.
- This measurement process is a true mathematical projection.
- Social voting process imposes this projection in order to construct a social choice.
- This quantum interpretation of such voting process clearly shows the violence of such kind of decision making.
- Disadvantage and dangers of such process have been demonstrated in France in 2002.

Conclusion

- Consequences of fractaquantum hypothesis:

  Mathematical formalism of quantum mechanics
  is supposed to have an extension to all "atoms" in Nature.

  The process of measuring has to be re-visited
  to all pairs (ℓ, B) of "atoms" with different scales.

- For classical election, the large scale imposes a direct
  generalization of the quantum measure process

  All the characteristics of the mathematical measure operator
  are controled by the large scale.

  Note the violence of a multiscale interaction
  through such a the measuring process.

- The mathematical framework of quantum mechanics
  for the measuring process has the potentiall to be adapted
  to generalized situations of two "atoms" of different scales.

Thank you!