Toward a Quantum Theory of Cognition: History, Development and Perspectives

Tomas Veloz[1,2,3]

tveloz@gmail.com

[1] Center Leo Apostel for Interdisciplinary Studies, Vrije Universiteit Brussel, Belgium

[2] Department of Mathematics, University of British Columbia Okanagan campus, 3333 University Way, Kelowna BC, V1V 1V7, CANADA

[3] Instituto de Filosofía y Ciencias de la Complejidad - IFICC. Los Alerces 3024 Ñuñoa, Chile.

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$$\mu(A) + \mu(B) - \mu(AB) \leq 1 \quad (2)$$

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  where $\mu(\cdot)$ represents the probability of an outcome, and AB is the outcome ‘(yes,yes)’of the joint experiment $E_A E_B$.
- Ex. $\mu(A) = 0.7$, $\mu(B) = 0.42$, and $\mu(AB) = 0.1$ is not a possible experience.

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  3. Bounds
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  6. **ETC.**
- However, there is a body of experimental evidence that challenges the validity of the conditions of possible experience in cognition

Example: Overextension of Conjunction

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- From a (Fuzzy) logical perspective, we expect that for all $x_k \in \Sigma$ the following holds:

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- However, psychological findings show strong 'overextensions' (inversions of (3)) in experimental data (Osherson & Smith, 1981; Hampton, 1988).

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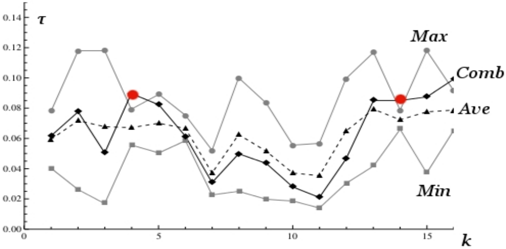

- We would expect $\mu(AB) \leq \mu(X)$, $X = A, B$. However, all objects are overextended (red points are *doubly* overextended)

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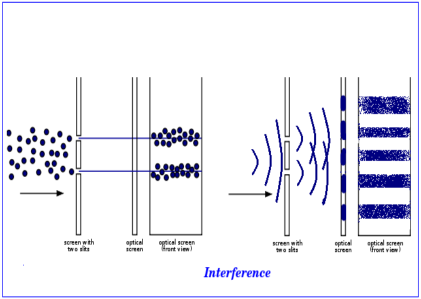
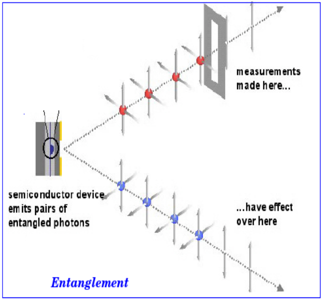
Violations of Possible Experience and Quantum Probability

- Quantum phenomena violate the conditions of possible experience because it is **contextual**, **non-compositional**, and **vague**





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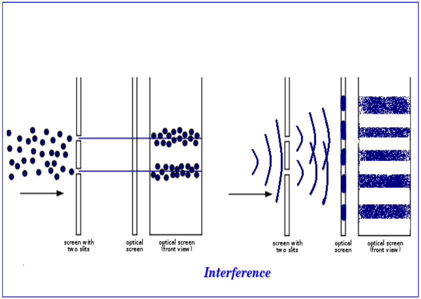

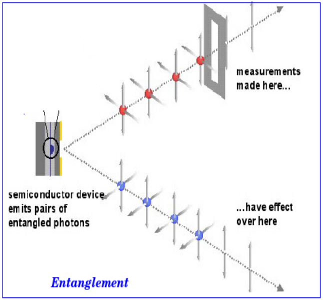

- The notion of possible experience strongly depend on the properties of the observation process...how is the cognitive observation process?
- Contextual, Non-compositional, Vague?...**Quantum?**

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- Quantum Cognition **does not** follow or take a position w.r.t. *Quantum brain* hypothesis

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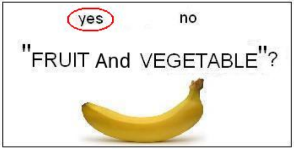
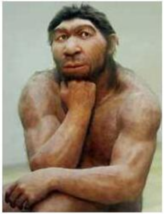
Modes of Thought

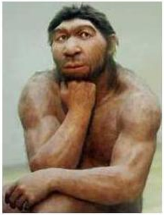
Emergent Mode of Thought (Hilbert Space)





YES

Logical Mode of Thought (Tensor Product)



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- **Second Mode (Classical):** The combination is a 'compositional' state $|AB_2\rangle = |A\rangle \otimes |B\rangle \in \mathcal{H} \otimes \mathcal{H}$ (can be an entangled state)

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$$|AB\rangle = \frac{ne^{i\phi}}{\sqrt{2}}(|AB_1\rangle + \sqrt{1-n^2}e^{i\theta}|AB_2\rangle$$

A Simple Model Illustrating the General Scheme

- The membership operator is $\mathbf{M}^F = \mathbf{M} \oplus (\mathbf{M} \otimes \mathbf{M})$. Then

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- Can we test if these **new conditions of possible experience** apply in other experiments in cognition?

Step Forward: Conjunctions and Negations of Concepts

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$$\begin{aligned}
I_A &= \mu(A) - \mu(AB) - \mu(A\bar{B}) = 0, && (5)\\
I_{\bar{A}} &= \mu(\bar{A}) - \mu(\bar{A}B) - \mu(\bar{A}\bar{B}) = 0, && (6)\\
I_B &= \mu(B) - \mu(AB) - \mu(\bar{A}B) = 0, && (7)\\
I_{\bar{B}} &= \mu(\bar{B}) - \mu(A\bar{B}) - \mu(\bar{A}\bar{B}) = 0, && (8)\\
I_{AB\bar{A}\bar{B}} &= 1 - \mu(AB) - \mu(\bar{A}B) - \mu(A\bar{B}) - \mu(\bar{A}\bar{B}) = 0. && (9)
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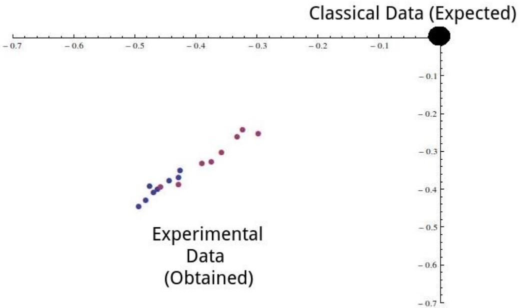

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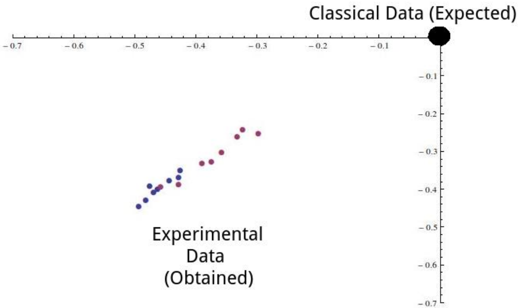

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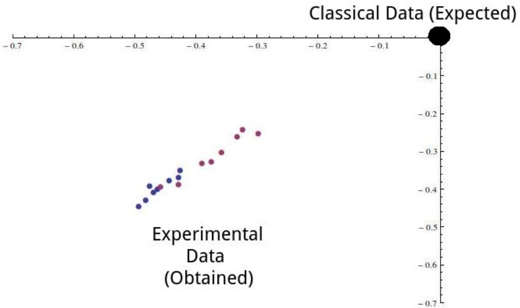

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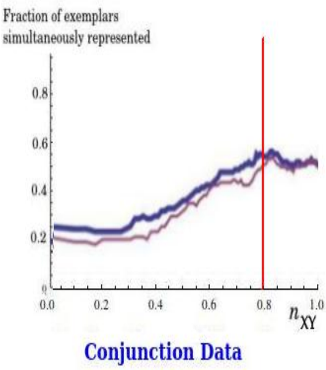
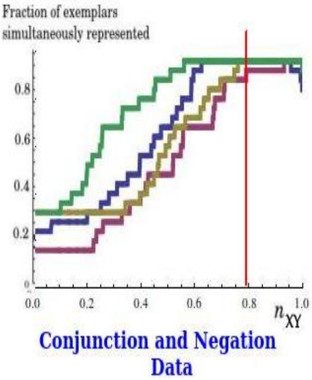
Comparing different datasets

Comparing the model's performance w.r.t data set on conjunction (Hampton), and on conjunction and negation (Aerts, Sozzo, Veloz)





Data compatible with Fock space model choosing $n_{XY} \sim 0.8$

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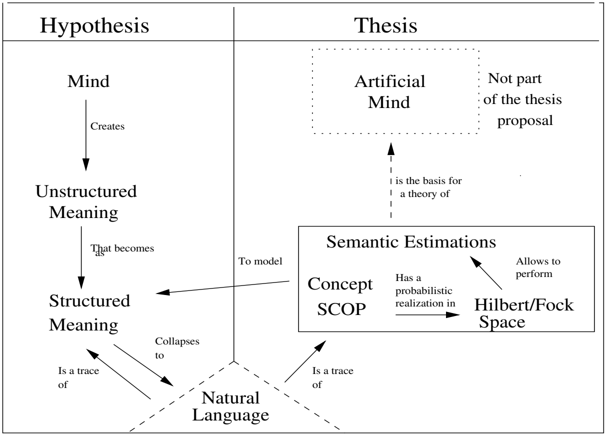
- Introduced modes of thought and a quantum-inspired framework for mind and language
- Quantum conditions of possible experience in cognition?

Thank you!...questions?

- Boole, G. On the Theory of Probabilities. Philosophical Transactions of the Royal Society of London (1962)
- Boole, G. The Laws of Thought. New York: Dover Edition (1958)
- Pitowsy, I. George Boole's Conditions of Possible Experience and the Quantum Puzzle. Brit. J. Phil. Sci. (1995)
- Van Rijsbergen, C. J. (2004). The geometry of information retrieval (Vol. 157). Cambridge: Cambridge University Press.
- Proceedings of the n-th Quantum Interaction Symposium-Qi-n, n=2007,...,2015.
- **These and the other references can be requested to me, space problems to put them all!**

Mind-Language and Quantum Cognition



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- Most people prefer 1 over 2, but 4 over 3! which contradicts STP.

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- It has been recently shown that the Machina Paradox (a more complex situation similar to the Ellsberg Paradox) is incompatible with all economic literature models, but the quantum approach still models it (Aerts et. al. 2013)