TI-games I: An Exploration of Type Indeterminacy in Strategic Decision-making

J. Busemeyer (Indiana University)
A. Lambert-Mogiliansky (PSE)

CNAM February 9 2009

Introduction

This paper belongs to a very recent and rapidly growing literature where formal tools of Quantum Mechanics are proposed to explain a variety of behavioral anomalies in social sciences and in psychology e.g., cognitive dissonance, framing effects, preference reversal, disjunction effect, inverse fallacy.

The use of quantum formalism in game theory was initiated by

- Eisert et al. (1999) who investigates games with quantum (mixed) strategies

- La Mura (2003) who investigates correlated equilibria with quantum signals in classical games.

⇒ In this paper we explore an extention of Type Indeterminacy-model (ALM, SZ, HZ) of decision-making* to strategic decisions.

- The TI-hypothesis extends the field of strategic interactions: actions impact not only on the payoffs but also on the profile of types, i.e., on who the players *are*.

- The TI-hypothesis gives new content to Harsanyi's approach. Fictitious Nature's move in Harsanyi's setting becomes a real move (a measurement). The theoretical multiplicity of types of a player becomes a substantial multiplicity of "selves".

- In the example we investigate, the TI-model can provide an explanation for why cheap talk promises can affect subsequent play. (Not a contribution to the literature on communication game!) .

Main findings

We find that there exists

- no distinction in terms of predictions between the standard Bayes-Harsanyi and Type Indeterminacy approaches in a simultaneous one-move context.

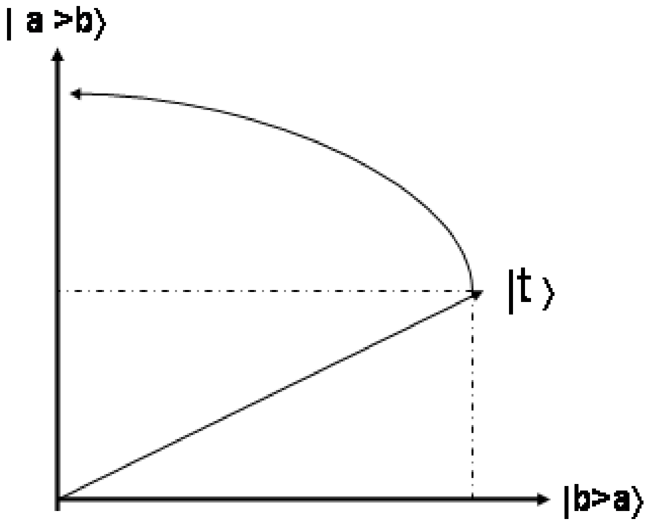
- but if
  - i. at least one player makes more than one move;
  - ii. those moves correspond to non-commuting Game Situations;

  $\Rightarrow$ Then a move with no informational content or payoff relevance can have impact on the outcome of the game.

The basic TI-model

- A simple decision situation is represented by an an operator, an *observable* called a *DS*.
- A DM is represented by his state or *type,* a vector $|t_i\rangle$ in a Hilbert space.
- The act of choosing (measuring the *DS*)⇔an operation on the type. It *changes the type* of the agent from being indeterminate to being fully or somehow determined wrt to the *DS*.
- The chosen item, actualizes an *eigentype* (or a *superposition)*. It is info about the preferences (type) of the agent as he is assumed to choose what he prefers.

One Choice experiment

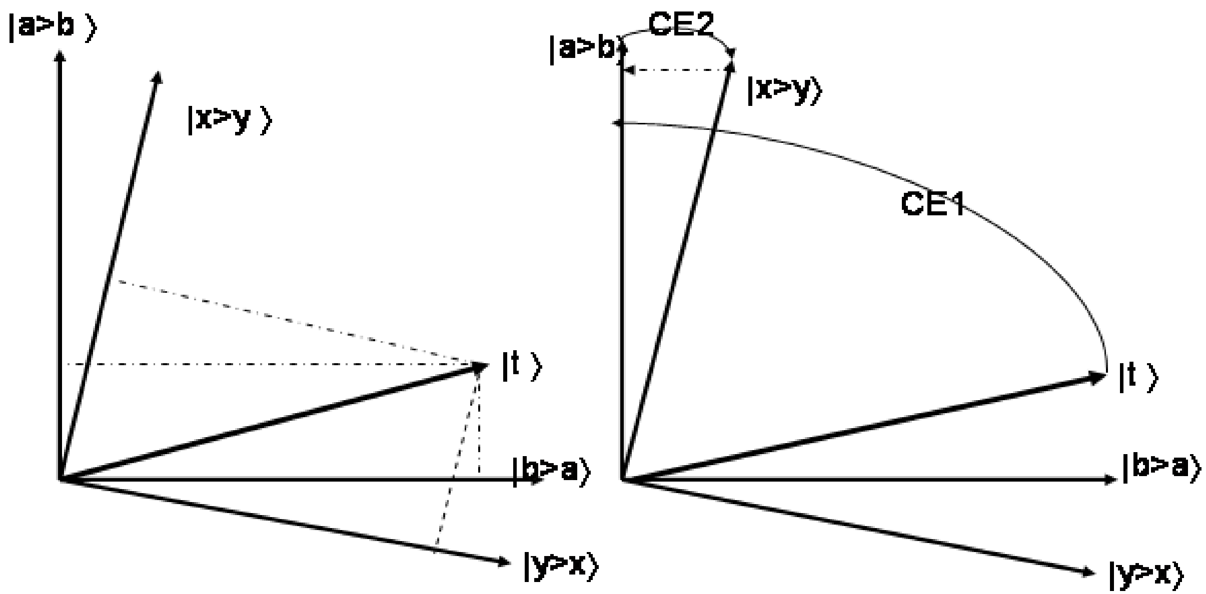


$||t\rangle = \lambda_1 \, |a>b\rangle \; + \lambda_2 |b>a\rangle$

$\text{Prob}(a; \, |t\rangle) = \lambda^2{}_1$

$\text{Prob}(a; \, |a>b\rangle \,) = 1$

Two non-commuting Choice experiments



CE1: Prob(a; |t ⟩) = $\lambda^2{}_1$ and |t > $\Rightarrow$|a>b⟩

CE2: x selected |a>b⟩ $\Rightarrow$ |x>y⟩ **so** Prob(a; |x>y⟩) < 1

TI-games

General features

- We denote by *GS* an observable that measures the type of a player in a game. The interpretation of the outcome is that the chosen action is a *best reply* against the opponent's expected* action.
- In any specific TI-game *M* we must distinguish between the type which is identified with the player and the *eigentypes* (selves) which is identified with the payoff functions in game *M*.
- The reasoning leading to the determination of the best-reply is performed at the level of the *eigentypes* of the game.

A single interaction

Consider a 2X2 symmetric game, *M*, we define the preference types of game *M* also called the *M*-eigentypes as follows:

θ_1 : prefers to cooperate whatever he expects the opponent to do (philantrope);

θ_2 : prefers to cooperate if he expects the opponent to cooperate with probability $p > q$ (for some $q \leq 1$) otherwise he prefers to defect (reciprocator);

θ_3 : prefers to defect whatever he expects the opponent to do ("PD-rational").

Let player 1 be described by the superposition:

$$|t_1\rangle = \lambda_1|\theta_1\rangle + \lambda_2|\theta_2\rangle + \lambda_3|\theta_3\rangle, \sum \lambda_i^2 = 1.$$

IF P1 plays with a $|t_2\rangle = |\theta_1\rangle$ with prob. $\lambda_1^2 + \lambda_2^2$ he plays *C* and (by L - v N))$|t_1\rangle \to |t_1'\rangle$

$$|t_1'\rangle = \frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_1\rangle + \frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}|\theta_2\rangle$$

with prob.λ_3^2 he plays D and $|t_1\rangle \to |\theta_3\rangle$. If $|t_2\rangle = |\theta_3\rangle$, $|t_1\rangle \to |\theta_1\rangle$ or $|t_1\rangle \to |t_1''\rangle = \frac{\lambda_2}{\sqrt{\lambda_2^2+\lambda_3^2}}|\theta_2\rangle + \frac{\lambda_3}{\sqrt{\lambda_2^2+\lambda_3^2}}|\theta_3\rangle$.

The *resulting type* depends on the opponent's type and corresponding expected play.

Formally no distinction from a classical info interpretation where the resulting type captures our revised beliefs about player 1.

Statement 1: The TI-model of a simultaneous one-move game is equivalent to a Bayes-Harsanyi model.

A multi-stage TI-game

The PD is preceded by game *N, a promise exchange game PEG* with a 3rd player. There are 3 eigentypes in PEG:

τ_1 : prefers to never make cheap-talk promises - let him be called the "honest type";

τ_2 : prefers to make a promise to cooperate if he believes the opponent cooperates with probability $p \geq q$ (in which case he cooperates whenever he is of type θ_2 or θ_1 or any superposition of the 2). Otherwise he makes no promises - let him be called the "sincere type";

τ_3 : prefers to promise that he will cooperate whatever he intends to do - he can be viewed as the "opportunistic type".

Information assumptions

i. All players know the statistical correlations (conditional probabilities) between the eigentypes of the two (non-commuting) games.

ii. At *step 2,* player 2 knows that player 1 has interacted with player 3 but he does not know the outcome of the interaction (action not observed).

We shall be interested in comparing predictions in protocol I (only PD) and at step 2 of protocol II (PD preceded by PEG).

In the classical setting we have the same outcome in protocol I and at *step 2* of protocol II. Follows from info assumptions.

The TI-model

Player 1's propensity to defect in protocol I

We assume that *M* and *N do not commute* with each other and we want to account for the indeterminacy due to PEG *NOT* played*:*

$$|t_1\rangle = \lambda_1'|\tau_1\rangle + \lambda_2'|\tau_2\rangle + \lambda_3'|\tau_3\rangle$$

Each one of the N–eigentype can in turn be expressed in terms of the eigentypes of game M :

$$\begin{aligned}
|\tau_1\rangle &= \delta_{11}|\theta_1\rangle + \delta_{12}|\theta_2\rangle + \delta_{13}|\theta_3\rangle \\
|\tau_2\rangle &= \delta_{21}|\theta_1\rangle + \delta_{22}|\theta_2\rangle + \delta_{23}|\theta_3\rangle \qquad \# \\
|\tau_3\rangle &= \delta_{31}|\theta_1\rangle + \delta_{32}|\theta_2\rangle + \delta_{33}|\theta_3\rangle
\end{aligned}$$

δ_{ij} are the elements of the basis transf. matrix (correlations). We assume $\delta_{13} = \delta_{31} = 0$.

We substitute for the τ_i and the initial type vector writes:

$$\begin{aligned}|t_1\rangle = {} & \lambda_1'(\delta_{11}|\theta_1\rangle + \delta_{12}|\theta_2\rangle + \delta_{13}|\theta_3\rangle) + \lambda_2'(\delta_{21}|\theta_1\rangle + \delta_{22}|\theta_2\rangle + \delta_{23}|\theta_3\rangle) \\ & + \lambda_3'(\delta_{31}|\theta_1\rangle + \delta_{32}|\theta_2\rangle + \delta_{33}|\theta_3\rangle).\end{aligned}$$

collecting the terms we obtain

$$\begin{aligned}|t_1\rangle = {} & (\lambda_1'\delta_{11} + \lambda_2'\delta_{21} + \lambda_3'\delta_{31})|\theta_1\rangle + (\lambda_1'\delta_{12} + \lambda_2'\delta_{22} + \lambda_3'\delta_{32})|\theta_2\rangle + \\ & (\lambda_1'\delta_{13} + \lambda_2'\delta_{23} + \lambda_3'\delta_{33})|\theta_3\rangle.\end{aligned}$$

In our numerical ex. $p(D\||t_1\rangle)_M = p(|\theta_3\rangle\||t_1\rangle)$:

$$p(D\||t_1\rangle)_M = (\lambda_2'\delta_{23} + \lambda_3'\delta_{33})^2 = \lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2 + \underline{2\lambda_2'\delta_{23}\lambda_3'\delta_{33}}$$

Protocol II

1. The Promise Exchange Game

We write P1's and P3's type vectors in terms of the eigentypes of game *N*:

$|t_1\rangle = \lambda_1'|\tau_1\rangle + \lambda_2'|\tau_2\rangle + \lambda_3'|\tau_3\rangle$ and $|t_3\rangle = \gamma_1'|\tau_1\rangle + \gamma_2'|\tau_2\rangle + \gamma_3'|\tau_3\rangle$.

We let player 1 play the PEG with "*tough" P3, i.e.,* $|t_1\rangle = |\theta_3\rangle$

- with probability $\lambda_1'^2 + \lambda_2'^2$ player 1 plays $no-P$ and collapses on

$$\left|\hat{t}_1\right\rangle = \frac{\lambda_1'}{\sqrt{(\lambda_1^{2\prime} + \lambda_2^{2\prime})}}|\tau_1\rangle + \frac{\lambda_2'}{\sqrt{(\lambda_1^{2\prime} + \lambda_2^{2\prime})}}|\tau_2\rangle$$

- With probability $\lambda_3'^2$ he collapses on $|\tau_3\rangle$. **We note that type τ_2 and τ_3 separate.**

2. *Player 1's propensity to defect in protocol II*

Having played PEG changed $|t_1\rangle$ into $\left|\hat{t}_1\right\rangle$ or $|\theta_3\rangle$ and $|\tau_2\rangle$, $|\tau_3\rangle$ separated

$$p(D\||t_1\rangle)_{MN} = p(no-P)p\left(D|\left|\hat{t}_1\right\rangle\right) + p(P)p(D\||\tau_3\rangle).$$

which gives

$$p(D\||t_1\rangle)_{MN} = \lambda_2^{2\prime}\delta_{23}^2 + \lambda_3^{2\prime}\delta_{33}^2$$

Comparing

$$p(D\||t_1\rangle)_M - p(D\||t_1\rangle)_{MN} = 2\lambda_2'\delta_{23}\lambda_3'\delta_{33} \qquad \#$$

Result 1: *When player 1 meets a tough player 3 at step 1, the probability for playing defect in the next following M game is not the same as in the M game alone,* $p(D\||t_1\rangle)_M - p(D\||t_1\rangle)_{MN} \neq 0.$

The soft player 3 case

Assume P3 is a θ_1 type. In the PEG τ_2 and τ_3 pool to promise. And when it comes to PD they interfere as in protocol 1 so

$$p(D\|t_1\rangle)_M = p(D\|t_1\rangle)_{MN}$$

There is NO effect of the promise stage.

Result 2

If player 1's move at step 1 does not separate between the N-eigentypes that would otherwise interfere in the determination of his play of D at step 2 then $p(D\|t_1\rangle)_M = p(D\|t_1\rangle)_{MN}$.

Intuition

- We know that τ_2 and τ_3 have a positive propensity to defect.
- When playing the PEG the two eigentypes may either separate or pool.
- When they separate (P1 is either $|t_1'\rangle$ or τ_3). The 2 selves τ_2 and τ_3 do not coexist in his mind any more and therefore they cannot interfere with each other.
- In contrast when they pool, the two selves are present in the resulting type $\left|\widehat{t}_1\right\rangle$ and they interfere in the determination of the best-reply in PD as in the case when no PEG is played.

Playing the PEG without any observation can have impact.

A possible explanation for the "cheap talk paradox" suggested by the example is "unobserved separation".

Reaching the promise response is more difficult (takes longer time) for the reciprocating type τ_2 than for the opportinistic τ_3. So the act of promise making separates between them. But that is not observed when they both choose *P*.

The probability to play cooperate increases after playing PEG with a tough P3, whenever $2\lambda_2'\delta_{23}\lambda_3'\delta_{33} < 0$. The interference term is negative: when the 2 selves interfere they tend to neutralize each other.

Examples of possible applications

- Impact of pre-play on the selection principle in multiple equilibria situations (risk dominance, payoff dominance, avoidance of loss etc...) Experiments show that a pre-play auction to play a coordination game tends to lead to loss avoidance.
- Selection of reference point in bargaining. (Winning a unrelated contest before playing a ultimatum game changes the offer and rejection threshold)
- The sunk cost fallacy: the act of buying a subscription to the theater impacts on your preferences so you go more often.
- Path dependence: a move can has far reaching consequences for subsequent play e.g., when it radically changes the type.

Concluding remarks

In this paper we have explored an extension of the Type Indeterminacy model of decision-making to strategic decision-making in a maximal information context.

- We first find that in a one-shot setting the TI-model is equivalent to the standard Bayes-Harsanyi approach to games of incomplete information.
- This is no longer true in a multi-stage setting. We give an example of circumstances under which the predictions of the two models are not the same.
- We show that the TI-model can provide an explanation for why an uninformative(unrelated) pre-play (cheap-talk promises) matter when standard theory predicts it does not.