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Heinz von Foerster, time and systems

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Introduction

Heinz von Foerster has been deeply interested by time and memory in the context of systems. In « Molecular ethology » he considers a linear dynamical system. Its output depends on the history of its state. He remarks that «…the failure of this simple machine to account for memory should not discourage one from contemplating it as a possible useful element in a system that remembers ». Independently, we proposed a linear dynamical system which may be seen as a modelisation of perception and memorization of time elapsed.

Leaving the domain of linearity, but along the line of Heinz von Foerster's interest for time and systems, we defined an internal time s, adapted to a dynamical system, different from the reference or external time t. For this purpose I consider internal duration $di(t_1,t_2)$ or internal time elapsed between two instants of reference t_1 and t_2. It generates an internal time $s = di(t_0,t)$, defined up to an additive constant. The choice of $di(t_1,t_2)$ depends upon the « weight » of the infinitesimal reference duration dt between t and t+dt. We propose that the internal duration, corresponding to the reference duration dt, is equal to $(dX(t)/dt)^2dt$, $X(t)$ being the state of the system at reference instant t. In a way $(dX(t)/dt)^2$ is an index of « importance » of instant t. As an example, we consider an « explosive-implosive » dynamical system. The corresponding internal time varies from 0 to a finite value while reference time varies from $-\infty$ to $+\infty$. Interpretations are proposed (physiology, cosmology).

1. Perception and memorization of time.

In « Molecular ethology » (Foerster,1970) Heinz von Foerster considers what he calls a « machine » that is to say a device with an input x and an output y, both x and y being functions of time. He points out that in the general case y(t) is not just a function of x(t) but a function of all the values taken by x from the begining (at t = 0 for example) to present instant t. In other terms y is obtained from x by application of an hereditary operator. A special case is given (x(t) and y(t) being real) by the solution of the linear differential equation

$$dy(t)/dt = ay(t) + x(t), \tag{1}$$

which is

$$y(t) = \exp(at) [y(0) + \int_{o,t} \exp(-au) x(u) du].$$

H. von Foerster remarks that the « course of events represented by x(u) is ‘integrated out’ » in the integral term but that nevertheless there is a « failure of this simple machine to account for memory », a fact that « should not discourage one from contemplating it as a possible useful element in a system that remembers ».

Independently I proposed (Vallée, 1977, 1986) a model which, even if it does not account for memory, considers memorization and perception and results from the « contemplation » of the following differential equation which is a slight generalisation of equation (1)

$$ds(t)/dt = -a(t) s(t) +b(t), \quad a(t) \geq 0, b(t)\geq 0, s(t_0) = 0, \ s(t) \text{ real}. \tag{2}$$

The solution of this equation is given by

$$s(t) = \int_{to,t} \exp(- \int_{u,t} a(v)dv) \ b(u)du. \tag{3}$$

The model described by (2) may be seen as representing perception and memorization of time elapsed since an initial instant t_o : t is a *reference time* and s(t) a *subjective time* as it is perceived and memorized.

If a(t) is identical to zero, there is no memorization but only pure perception. We have

$$s(t) = \int_{to,t} b(u)du. \tag{4}$$

reference duration dt is perceived subjectively as being ds = b(t)dt. In other words b(t) may be seen as the *weight of instant* t. If b(t) = b/t we have

$$s(t) = b (\text{Log}t - \text{Log}t_o).$$

So $s(t)$ tends to infinity when t_o tends to zero. While reference time t varies from 0 to $+\infty$, subjective time varies from $-\infty$ to $+\infty$. This subjective time may be compared to Milne's cosmological time (Milne, 1948).

If $a(t)$ is not identical to zero we have, as seen above,

$$s(t) = \int_{to,t} \exp(- \int_{u,t} a(v)dv)\, b(u)du. \qquad (3)$$

We may write, from (2),

$$ds(t) = (- a(t)\, s(t) + b(t))\, dt.$$

The *weight of instant* t is not given only by $b(t)$, the *degree of attention* given to instant t. A *factor of oblivion* $a(t)\, s(t)$ must be substracted, diminishing the intensity of memorization of past instants.

2. Internal time of a dynamical system

Along the line of Heinz von Foerster's interest for time and systems, we want now to give a definition of the *internal time*, or intrinsic time, of a dynamical system (Vallée, 1996, 2005) evolving independently of its environment. The notion of *internal time* s is opposed to that of *reference time* t or external time, taken for granted, wich is used in the evolution equation. The basic idea is that the internal time does not elapse if the state of the system does not change, a conception close to that of Aristotle and also of Augustine, considering that time ceases to be known when the « soul » does not vary. More generally we consider that internal time elapses more quickly, making duration longer, when the state of the system changes rapidly.

So if $X(t)$, belonging to a finite dimensional linear space, is the state of the system at *reference instant* t, any positive and increasing function, null for $t = 0$, of a norm of $dX(t)/dt$, is a measure of the intensity of change of the system at instant t. We make a simple choice, that of the square of the euclidian norm, or scalar square, $(dX(t)/dt)^2$ which may be seen as an index of « importance » of instant t. So we consider that the *internal duration* corresponding to *reference duration* dt, between t and t+dt, is equal to $(dX(t)/dt)^2$ and we define (Vallée, 1996, 2005) the *internal duration* $d_i(t_1,t_2)$ of interval (t_1,t_2), whose *reference duration* is t_2-t_1), by

$$d_i(t_1,t_2) = \int_{t1,t2} (d(X(t)/dt)^2\, dt\, . \qquad (5)$$

So if $(d(X(t)/dt)^2$ is equal to 0 the interval, the internal duration is 0 and if $(d(X(t)/dt)^2$ is equal to 1, the internal duration is equal to the reference duration. In short the higher the values of $(d(X(t)/dt)^2$ on the interval, the longer the internal duration. We can now define the *internal time* s as a function of t

$$s(t) = d_i(t_0,t) = \int_{to,t} (dX(u)/du)^2\, du, \qquad (6)$$

where t_0 is any reference instant. So s(t) is defined up to an arbitrary additive constant. Of course we have

$$d_i(t_1,t_2) = s(t_2)-s(t_1).$$

2.1 Example

We call *explosion* the evolution of a system whose state vector has a modulus starting with value 0 at t = 0 then increasing with t, and such that the modulus of its speed vector starts with value $+\infty$ at $t = 0$. The first instants of the evolution of the system have an exceptional importance since, with the Euclidian norm, the scalar square $(d(X(t)/dt)^2$ of the speed vector tends to $+\infty$ when t tends to 0. On the contrary we have an *implosion* when X(t) decreases with t and attains value 0 at the final instant while $(d(X(t)/dt)^2$ tends to $+\infty$. A system may be explosive at the begining and implosive at the end, then we say that we have an *explosion-implosion*.

We consider now an *explosion-implosion* whose equation of evolution, where for the sake of simplicity X(t) is a scalar, is given by (Vallée, 1996, 2005)

$$dX(t)/dt = \mathrm{sgn}(p-t)\,(q/p)\,(q^2- X^2(t))^{1/2}/X(t),\ \ X(0) = 0,\ p \text{ and } q > 0,\ \ 0 \leq t \leq 2p, \qquad (7)$$

sgn(p-t) being the sign of p-t. The solution of this equation is

$$X(t) = (q/p)\,(p^2- (p-t)^2)^{1/2}. \qquad (8)$$

The graph of function X(t) is an half-ellipse of great axis 2p and small axis 2q.When t varies from 0 to 2p, X(t) increases from 0 to q then decreases from q to 0, with a speed of infinite absolute value at 0 and 2p. The square of the speed of evolution is given by

$$(dX(t)/dt)^2 = (q^2/p^2)\,(p-t)^2/t(2p-t) = (q^2/2p)\,(1/t - 2/p + 1/2p-t)$$

and so the index of « importance » of instant t is infinite both at the begining ($t = 0$) and at the end ($t = 2p$). If we integrate $(dX(t)/dt)^2$ from t_1 to t_2 we obtain the *internal duration* of the reference time interval (t_1,t_2)

$$d_i(t_1,t_2) = (q^2/p)\,(\mathrm{Log}(t_2/t_1) - 2(t_2-t_1)/p - \mathrm{Log}(2p-t_2/2p-t_1)). \qquad (9)$$

If we choose $t_0 = p$, we have a *reference time* s ,defined up to an additive constant, by

$$s(t) = d_i(p,t) = q^2/2p\,(\mathrm{Log}t - 2(t-p)/p - \mathrm{Log}(2p-t)). \qquad (10)$$

So, when the *reference time* t varies from 0 to 2p, generating a finite *reference duration* of the evolution equal to 2p, the *internal time* s varies from $-\infty$to $+\infty$, generating an infinite *internal duration* of the evolution of the explosive-implosive system. The initial instant 0 is pushed back to $-\infty$ and the final instant 2p is pushed forward to $+\infty$. The internal duration of any interval (0,t) is infinite as well as the internal duration of any interval (t,2p).

2.2 Interpretations

In the first interpretation we consider an *explosion-implosion* as the evolution of a living being whose birth may be compared to a kind of explosion and the end of life as an involution or a kind of implosion. In our model the explosive and the implosive parts are symmetical. This is not very realistic in the present case, but may be accepted if we consider only the qualitative aspects. From the *internal time* point of view the initial instant is pushed back to ∞ and the final instant is pushed forward to $+\infty$ (Vallée, 1996, 2005). The last proposition is obviously questionable. If we consider the limit case of our example obtained when p and q tend to infiniy, while q^2/p remains equal to a constant h , the explosion-implosion becomes a mere explosion, represented by a parabola, and $s(t) = h\,\mathrm{Log}t$. We obtain the logarithmic time proposed by Lecomte du Nouÿ (1936) as a physiological time and also by Heinz von Foerster (2002).

The concept of in*ternal time* may be interpreted in cosmology. The equation of evolution of the universe, whose state at instant t is supposed to be given by its mere radius R(t), is given, in a rather simple case by

$$(d(R(t)/dt)^2 = ((8\quad G / 3) \ / R^2(t)) - c^2, \ R(0) = 0, \qquad (11)$$

G being the gravitational constant, c the speed of light, a positive constant. We consider the *explosion-implosion* whose equation of evolution, seen above, is

$$dX(t)/dt = \mathrm{sgn}(p-t)\ q/p\ (q^2-X^2(t))^{1/2} / X(t). \qquad (7)$$

Taking the square of the two members of (7), we have

$$(d(X(t)/dt)^2 = (q^2/p^2)(q^2-X^2(t)) / X^2(t) = (q^4/p^2)/ X^2(t) - q^2/p^2.$$

This equation is identical to equation (11) if we replace X(t) by R(t) and write

$$p = (8 \quad G / 3)^{1/2} / c^2, \quad q = cp = (8 \quad G / 3)^{1/2}/c \qquad (13)$$

The *internal time* of this system, which we propose to call *generalized cosmological time,* is given, according to (10), by (Vallée 1996, 2005)

$$s(t) = (pc^2/2)\ (\mathrm{Log}t - 2(t-p)/.p - \mathrm{Log}(2p-t)), \qquad (14)$$

p having the value given by (13). The initial reference instant t = 0 (big bang) is pushed back to-∞ and the final reference instant t = 2p (big crunch) is pushed forward to +∞. If we consider instants very far from the big crunch (t negligible compared to p) we obtain a logarithmic time as proposed by Milne (1948) under the name of cosmological time.

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